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On cooling tea and coffee

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Factors influencing the rate of cooling of hot coffee and tea have been investigated theoretically and studied experimentally using deliberately "domestic" apparatus. It is demonstrated that black coffee cools faster than white coffee under the same conditions. Under most (but not all) circumstances, if coffee is required to be as hot as possible several minutes after its preparation, any milk or cream should be added immediately, rather than just before drinking.

I. INTRODUCTION

You have just made a cup of coffee (or tea), which you intend to drink white. However, you are called away and so prevented from drinking it for several minutes. Assuming that you wish the coffee to be as hot as possible when you return, when should you add the milk—as soon as the coffee is prepared (scheme 1) or just before drinking it (scheme 2)? This question is hardly new, but we present here a simple realistic model and some actual measurements relevant to the problem, and discuss their practical significance.

Two opposing thermodynamic effects operate. When the coffee is hottest it loses heat more rapidly (as predicted by Newton's law of cooling). If this effect operated alone, we should of course add the (cold) milk as soon as possible in order to reduce the total heat loss during the cooling period. However, the addition of the milk cools the coffee directly, and the temperature reduction given by a simple law of mixtures is clearly greater the greater the initial temperature of the coffee. This effect operating in isolation would dictate deferring the addition of the milk to the last possible moment. It is necessary to make a mathematical analysis to determine which effect is more important.

II. THEORETICAL ANALYSIS

Newton's law of cooling states that, at least for small values of the excess temperature of a body relative to its surroundings (ΔT), the rate of cooling of a hot body is proportional to ΔT. The work of Dulong and Petit, and of Langmuir, led to the Lorentz cooling law, in which the rate of cooling is proportional to $(\Delta T)^{5/4}$. There is no contradiction, however, because the cooling law depends on the environmental conditions. The Lorentz law corresponds to the case of natural convection, whereas the Newtonian law corresponds to forced convection. The Newtonian law is thus more appropriate under normal circumstances[1]; this is fortunate, because it is rather easier to integrate than the Lorentzian formula.

Since the heat content of a body (in the absence of changes of phase) is linearly dependent on its temperature, we may express Newton's law as

$$\frac{d(\Delta T)}{dt} = -\frac{\Delta T}{\tau}. \qquad (1)$$

The solution to this differential equation is

$$\Delta T = \Delta T^{(0)} \exp(-t/\tau), \qquad (2)$$

i.e., the temperature relaxes exponentially toward ambient with a characteristic time constant (τ) that depends upon the heat capacity of the body, among other factors.

Suppose that black coffee cools with a time constant τ_B and white coffee with a time constant τ_W. We shall assume these figures to refer to equal volumes of coffee. While in practice the volume of white coffee will frequently be somewhat greater, the effect of this difference is small, as discussed below. We shall further suppose that black coffee and milk have the same volume heat capacity, which will be close to that of water. Again, the effect of this approximation upon our conclusions will be small. With this simplification, the heat content of a volume of either fluid is proportional to its volume multiplied by its temperature (plus a constant).

Let the temperatures of black coffee and milk exceed ambient by ΔT_c and ΔT_m, respectively. If ν volumes of milk are added to 1 volume of black coffee, the resulting temperature excess (ΔT_w) of the white coffee is

$$\Delta T_w = (\Delta T_c + \nu \Delta T_m)/(1+\nu). \qquad (3)$$

According to the first scheme proposed, the milk is added immediately, and the resulting white coffee is allowed to cool for a time t. If the initial temperature excess of the (black) coffee is $\Delta T_c^{(0)}$, this will be reduced, on adding the milk, to

$$(\Delta T_c^{(0)} + \nu \Delta T_m)/(1+\nu) . \quad (4)$$

After cooling for a time t, this temperature excess will have fallen to

$$\Delta T^{(1)} = [(\Delta T_c^{(0)} + \nu \Delta T_m)\exp(-t/\tau_W)]/(1+\nu) . \quad (5)$$

In the second scheme, the black coffee is first allowed to cool for the time t. Its temperature excess will fall during this period from $\Delta T_c^{(0)}$ to $\Delta T_c^{(0)}\exp(-t/\tau_B)$. Upon adding the milk, which we shall assume has been maintained at the excess temperature ΔT_m, the temperature of the mixture will fall to

$$\Delta T^{(2)} = [\Delta T_c^{(0)} \exp(-t/\tau_B) + \nu \Delta T_m]/(1+\nu) . \quad (6)$$

The problem thus resolves itself into that of finding which of $\Delta T^{(1)}$ and $\Delta T^{(2)}$ is greater. The usual solution involves assuming that $\tau_B = \tau_W = \tau_0$. In this case, it may readily be shown that

$$\Delta T^{(1)}/\Delta T^{(2)} = (1 + \nu \Delta T_m/\Delta T_c^{(0)})/[1 + (\nu \Delta T_m/\Delta T_c^{(0)})\exp(t/\tau_0)] . \quad (7)$$

If $\Delta T_m > 0$ (i.e., the milk is above ambient temperature), this expression will be less than unity and so scheme (2) *must* result in hotter coffee. If, on the other hand, $\Delta T_m < 0$ (as it will be if the milk is kept in a refrigerator), the expression will be greater than unity and scheme (1) results in hotter coffee. Indeed, this seems intuitive—if the milk is warm, add it later, and if it is cold add it straight away. However, if we now consider the case of $\tau_B < \tau_W$, which as we shall see in Sec. III is the case in practice, the situation becomes more complicated. The ratio $\Delta T^{(1)}/\Delta T^{(2)}$ is now

$$\Delta T^{(1)}/\Delta T^{(2)} = [(1 + \nu \Delta T_m/\Delta T_c)\exp(t/\tau_B - t/\tau_W)]/[1 + (\nu \Delta T_m/\Delta T_c)\exp(t/\tau_B)] . \quad (8)$$

If $\Delta T_m < 0$, this expression will always be greater than unity, and scheme (1) again wins. On the other hand, if $\Delta T_m > 0$, the better scheme will depend on the value of $(\Delta T_c/\nu \Delta T_m)$, on the time t, and on the two time constants.

When $\Delta T^{(1)} = \Delta T^{(2)}$, the above expression can be rearranged to give

$$\Delta T_c/\nu \Delta T_m = [1 - \exp(-t/\tau_W)]/[\exp(-t/\tau_W) - \exp(-t/\tau_B)] = f(t) . \quad (9)$$

It therefore follows that scheme (1) produces hotter coffee so long as $(\Delta T_c/\nu \Delta T_m) > f(t)$, otherwise scheme (2) is better. Thus, in the realistic case of $\tau_B < \tau_W$, if the milk is cold it is still best to put it in straight away but, if it is warmer than the ambient temperature, the best time to put it in depends on the time for which the coffee is to be left to cool.

III. EXPERIMENTAL OBSERVATIONS

In order to investigate the aptness of the foregoing analysis, and to try to gain a general understanding of the significant influences on coffee cooling, we have made various measurements of the cooling of black coffee, white coffee, and a number of other liquids. The experimental hardware consisted of a cylindrical glazed china mug, predominantly white in color, resting on sufficient insulation that heat loss through the base could be neglected. The mug had an inside diameter of 70 mm and a capacity of 320 ml, although it was normally filled with only 250 ml of liquid. Temperatures were measured using a standard -10 to $+110\,^\circ$C mercury-in-glass thermometer, which we estimated we could read to an accuracy of 0.3 °C. The thermometer rested with its bulb on the bottom of the mug during the cooling process, although immediately before making each measurement (once a minute) it was raised to the middle of the mug, used for stirring briefly and gently, read, and replaced. Our observations suggested that the precise method by which the temperature measurements were made did not significantly affect the deduced value of the cooling time.

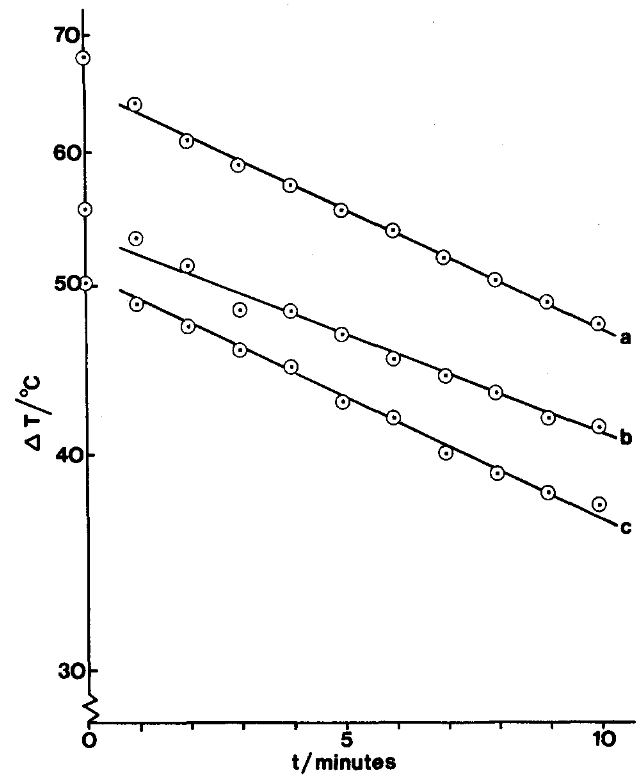

Fig. 1. Typical cooling curves, plotted as (log ΔT) vs time: (a) black coffee ($\tau_B = 32$ min); (b) white coffee ($\tau_W = 38$ min); (c) hot water, under different insulation conditions ($\tau = 33$ min). The straight lines were drawn by least-squares regression, ignoring the first data point in each case.

The black coffee used in this work was prepared using a leading brand of instant coffee granules. The volume ratio ν of milk (ordinary pasteurized and homogenized cow's milk) added was 0.1; i.e., the white coffee consisted of 227 ml black coffee, with 23 ml milk stirred into it. Observations were made of the temperature excess ΔT, as a function of the time t, for a period of 20 min. Figures 1(a) and (b) show the results for black and white coffee during the first 10 min, plotted on a log-linear graph for identical experimental conditions. It can be seen that Newton's law of cooling is obeyed; indeed, within the experimental accuracy of the observations, we found no deviation from the predicted exponential decrease throughout the whole 20-min period, apart from a slightly accelerated cooling rate during the first minute. The cooling times τ were calculated, using a least-squares regression analysis and ignoring the first minute's data in each case, to be 31.7 ± 0.3 min for black coffee and 38.3 ± 1.5 min for white coffee. This observed difference of 7 ± 1 min—i.e., nearly 20%—was found to be reproducible when changes were made to the thermal insulation of the mug.

We attempted further investigation to try to explain this difference in the cooling rate. The hypotheses that we attempted to test were that the difference might be caused by slight inaccuracies in measuring out the quantity of liquid; by differences in the blackbody radiative efficiencies; and by a difference in viscosity (which would influence the internal convection rate).

To test the theory that the cooling rates were influenced by the total volume of liquid, we performed similar experiments with hot water, first filling a mug (250 ml) and then half filling it. The cooling times were found to be 40 ± 1.5 min and 28.0 ± 0.7 min, respectively. This difference is not surprising in view of the much larger surface-to-volume ratio in the latter case, but it suggests that a variation of a couple of percent in the volume of liquid contained in the mug would cause no greater variation in the cooling time. This is evidently inadequate to explain the measured difference of about 20% between black and white coffee.

To investigate the possibility that the differences in cooling rates might be caused by differences in blackbody radiative efficiency, we repeated the original experiment with hemispherical domes of black paper or of aluminum foil surrounding the mugs. These domes had radii of about 150 mm, and had small holes at the top to allow the thermometer to pass through. They were shaped around a large mixing bowl, to ensure a consistent size. If the difference in cooling rate is attributable to greater blackbody radiation from black coffee, the dome of aluminum foil should have entirely removed the difference (since aluminum reflects practically all of the radiation incident upon it near the wavelength of maximum radiation, about 8 μm for a body at 70 °C), whereas the dome of black paper (an absorber) should have accentuated it. In fact, we found cooling times of 50.1 ± 0.7 min for black coffee and $56.3 + 0.8$ min for white coffee with the aluminum dome, and 41.2 ± 0.5 and 48.6 ± 0.6 min, respectively, for black and white coffee with the paper dome. These figures suggest that the *difference* in cooling time is virtually independent of the radiation loss, although clearly radiation is a significant influence since the cooling times were longer under the aluminum dome than under the paper dome. The fact that all of these cooling times were longer than in the absence of any dome may be attributed to the retention of warm air by the domes.

To assess the likely effect of viscosity, we measured the cooling time for "Golden Syrup" (molasses). This is highly viscous, even at 90 °C. Although the measured cooling time, 55 ± 3 min, was indeed longer than observed for other liquids, the change did not appear to be great enough for the difference between black and white coffee to be ascribed to (small) viscosity differences.

Finally, although this has no direct bearing on the *difference* between the cooling rates of black and white coffee, we investigated the effect of a draught of air over the surface of the cooling liquid. The draughts were provided by small electric fans and, even though precautions were taken to ensure that the mug and the surface of the liquid itself were protected from the flow of air, the effect was found to be dramatic. For coffee (black or white) or water, the cooling time was reduced by about 40% for a draught of about 3 m/s, and by about 60% for a draught of about 10 m/s.

IV. DISCUSSION

The principal observation made in this work is that black coffee cools significantly faster than white coffee, by about 20% under normal conditions. Various influences on the cooling rate have been investigated.

A. Factors that affect black and white coffee equally

Not surprisingly, the cooling time was found to be approximately proportional to the ratio of volume to total surface area of the liquid, other things being equal. Thus, for a cylindrical container of (constant) radius r filled to a depth h,

$$\tau \propto h/(h+r)\,. \qquad (10)$$

If $h = 2r$ (approximately true in practice),

$$\Delta\tau/\tau \sim \Delta h/3h\,. \qquad (11)$$

Thus the fact that a cup of white coffee is typically about 10% more full than the cup of black coffee from which it is prepared—although the two volumes were practically equal in our experiments—suggests that the white coffee should cool only about 3% slower. This predicted effect is small compared to the 20% difference in cooling times that we found for equal volumes of black and white coffee. Even if the suggested variation τ is proportional to volume-to-surface ratio is not strictly correct, the result (11) above should not be significantly in error.

The effect of varying the insulation below the mug (in the form of poorly conducting mats, piles of paper, and so on) was to change cooling times by no more than 5%. We took care always to compare cooling rates under the same insulation conditions; in any case, it is clear that differences in insulation would be unlikely to mask the intrinsic difference in rates.

Finally, we note that, of all effects likely to act equally on either liquid, that of a draught of air is greatest. A draught of only a few meters per second was sufficient to reduce the cooling time by 40%, and this is clearly sufficient to mask the intrinsic difference between black and white coffee. For this reason, all of our comparative experiments were performed in still air.

B. Factors that might affect black and white coffee differently

Our experimental investigation of the effect of blackbody radiation suggests that there is virtually no difference in radiative efficiency between the two liquids. Indeed, this is consistent with the fact that black and white pigments used in paints have very similar (and high) thermal emissivities. In other words, the color, which is, of course, determined by emissivity at visible wavelengths, bears no simple relationship to the total emissivity.[2] We might expect a similar relationship to hold for coffee.[3]

It also seems unlikely that a difference in viscosity (which influences the internal convection) can account for the difference in cooling times, since the viscosities of black and white coffee are not markedly different; also syrup (molasses), which has a viscosity several orders of magnitude greater, cools only twice as slowly.

The mechanism that we conjecture to be responsible for

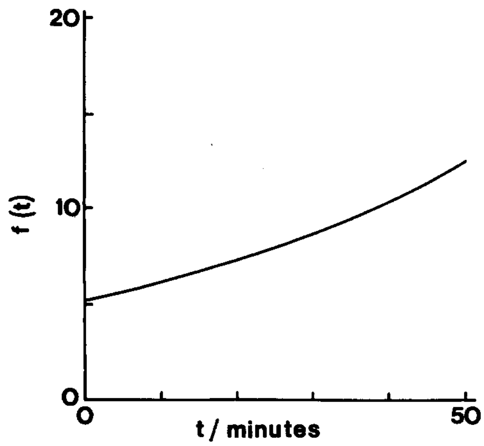

Fig. 2. Plot of $f(t)$, assuming $\tau_B = 32$ min and $\tau_W = 38$ min.

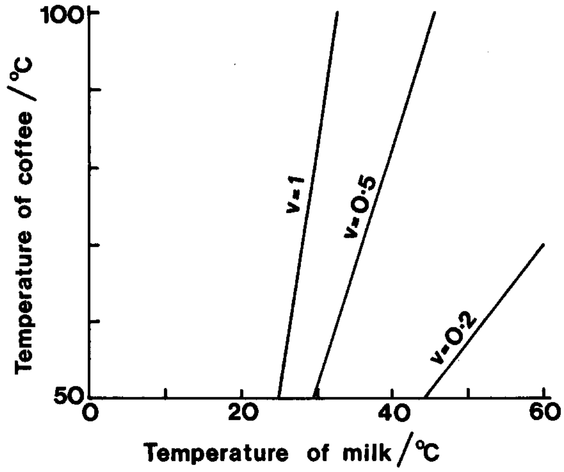

Fig. 3. Figure illustrating the best time to add the milk to the coffee, using the curve of $f(t)$ in Fig. 2 and assuming an ambient temperature of 20 °C and that the coffee is left to cool for 10 min. Here, ν is the relative volume of milk added to 1 volume of black coffee. If the point representing the temperatures of the coffee and milk lies to the left of the relevant line, the milk should be put in immediately; if to the right, at the end of the 10-min cooling period.

the difference in cooling times is a reduction in evaporation rate when milk is present in the coffee, but we were unable to perform a suitable experiment, with our deliberately "domestic" approach, to confirm this.

C. When to add the milk

Using the theory developed in Sec. II, and our measured values of $\tau_B = 32$ min and $\tau_W = 38$ min, we may evaluate $f(t)$. The result is shown in Fig. 2. If we assume that $\Delta T_c = 70$ °C, $\Delta T_m = 30$ °C, and $\nu = 0.2$, then $(\Delta T_c / \nu \Delta T_m) = 11.7$, for which value of $f(t)$ the time required is about 46.5 min. Thus the resultant coffee will be hotter if the milk is put in last only if it is left to cool for more than 46.5 min. However, the coffee will then be rather cool, at only 18 °C above ambient.

If we assume that the longest time for which one might wish to leave the coffee is 10 min, then the critical value of $\Delta T_c / \nu \Delta T_m$ is about 6 (from Fig. 2). Thus if $\Delta T_c = 70$ °C, it can be seen that the effect of the difference between the two time constants will only be important if ν is large. For example, if $\nu = 0.5$, milk, which is less than 23 °C above ambient, should be put in immediately whereas warmer milk should be put in as late as possible. This is illustrated graphically in Fig. 3. If the excess temperature of the milk is 35 °C, the difference in drinking temperature amounts to nearly 1 °C.

D. Factors omitted from our analysis

The analysis presented here omits a number of points. Walker[4] has drawn attention to the cooling produced by the dissolution of sugar (for those who take it), and by vigorous stirring, which brings cool liquid to the surface faster than convection alone would do. He also points to the effect of leaving a metal spoon in the mug—which will absorb, radiate, and conduct heat from the coffee—and of the color of the mug itself. He has also investigated the effect of stirring cream into hot water, and of floating (whipped) cream on the surface.[5] He plots temperature as a function of time for these liquids but, as he does not quote the ambient temperature, it is not possible to make a precise estimate of the cooling times. However, if the ambient temperature is estimated as 20 °C, his data give cooling times of 30 min for hot water, 25 min for hot water with cream stirred in, and 43 min for hot water with the cream floating on top. The difference between the last two figures is presumably accounted for by the insulating effect of the layer of cream, as well as the prevention of evaporation. Smith[1] has analyzed the behavior of such a 'Gaelic coffee' system and demonstrated that it indeed invariably results in significantly hotter coffee than is produced by stirring in the cream.

V. CONCLUSIONS

The most significant conclusion to follow from this work is that black coffee cools faster than an equal volume of white coffee, by about 20%. The cooling in both cases obeys Newton's law. This difference is not contributed to significantly by a difference in blackbody radiative efficiency and we suspect that the dominant factor is that of milk reducing the rate of evaporation.

If black coffee is prepared in a mug and is to be drunk white and as hot as possible some minutes later, the optimum time at which the milk should be added depends on its temperature. If the initial milk temperature is below ambient, it should be added straight away. If it is initially above ambient, the time depends on the volume fraction of milk to be added, on its excess temperature, and on the total time interval between making the coffee and drinking it.

This work has also illustrated cooling rates that result from a partially filled mug, a draught, and the absence of good thermal insulation below the mug.

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