Wind Energy : Useful pictures
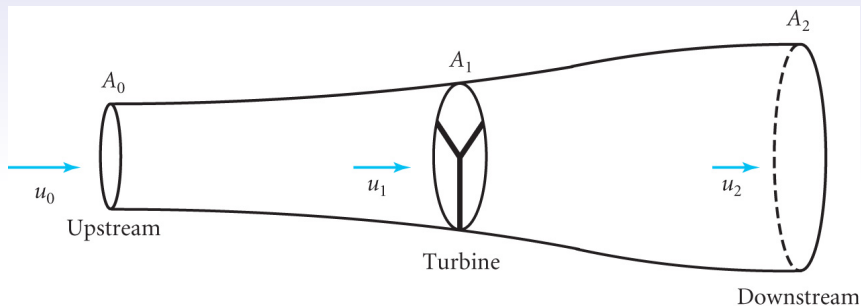
Wind Energy : Derivation of Betz Limit

Figure: Speeds and areas for stream tube in Betz derivation

▶ Fig. 1 is useful for derivation of Betz limit. One finds

$$u_1 = \frac{(u_0 + u_2)}{2}$$

by comparing two formulae for the power transferred.

▶ After extremising the power, we learn

$$u_1 = \frac{2}{3}u_0$$

and using the above

$$u_2 = \frac{1}{3}u_0$$
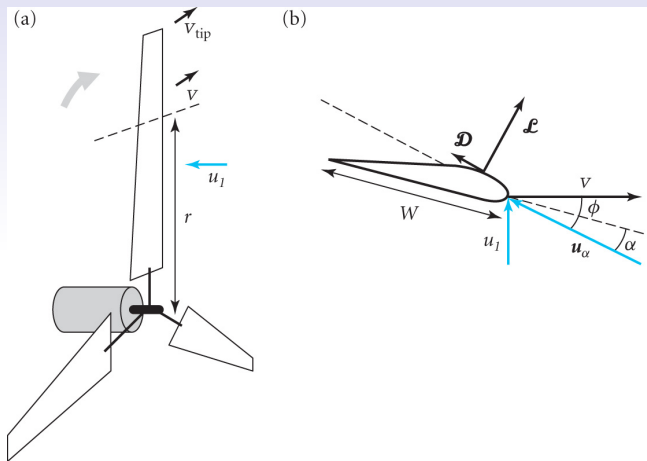
Wind turbine and cross-section

Figure: Cross-section of turbine and angles

- Fig. 2 (from Andrews-Jelley) is useful for the geometry leading to

$$tan\phi = \frac{2R}{3r\lambda}$$

- This equation is used in understanding twist of the aerofoil x-section along the turbine blade.
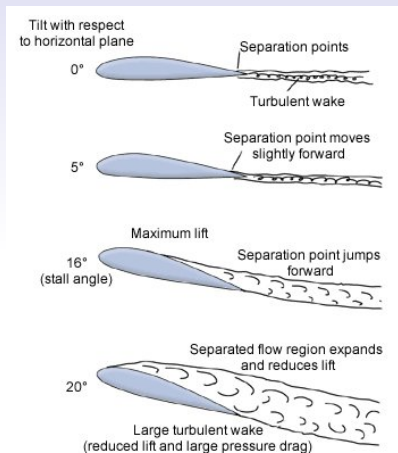
Angle of attack and fluid flow

Figure: Angle of attack

- ▶ Figure shows that the wind flow around an aerofoil (x-section of turbine) depends on angle of attack.
- ▶ There is an optimum angle of attack, typically a few degrees.
- ▶ See you-tube :
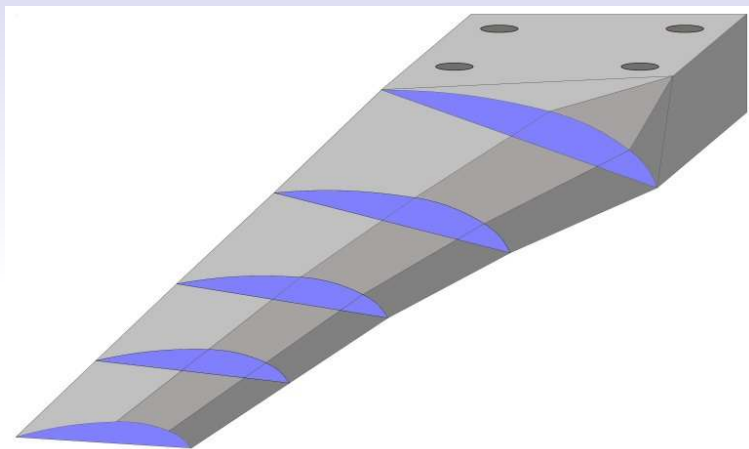 "how wings work? Smoke streamlines around an airfoil"
  http://www.youtube.com/watch?v=6UlsArvbTeo

Practical Meaning of the twist

Figure: Meaning of the twist

This figure is from
http://www.thebackshed.com/windmill/PropellerBlades.asp

Practical Meaning of the twist

Other applications of the importance of "angle of attack" – in particular, stall that occurs at large angles. In turbine control.

Turbine control : By adjusting the angle in real time, for high wind speeds, one can reduce the lift force so the turbine does not turn at high speed abd cause large stresses on the turbine.

In aviation, knowing about stall is an essential part of pilot training.
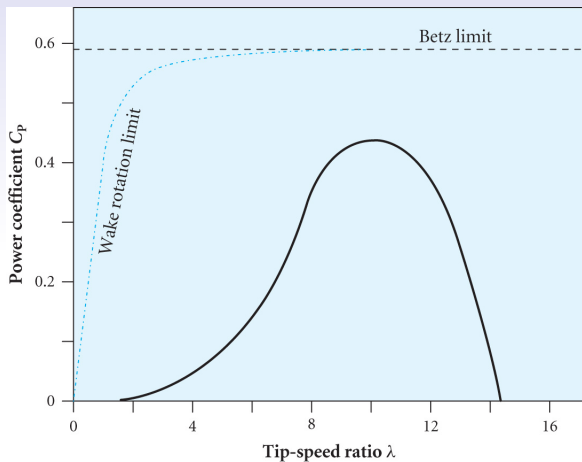
The power-coefficient versus λ curve

Figure: From AJ

The power-coefficient versus λ curve

The derivation of Betz limit only used linear momentum transfer and energy momentum conservation.

More realistic calculations, have to take into account, for example, angular momentum exchanged between wind turbine and air. Outgoing air also has some angular momentum.

These more detailed calculations help understand the power curve shown above.

These power curves are used in specifications of commercial wind turbines. Can be also be determined by measurements.

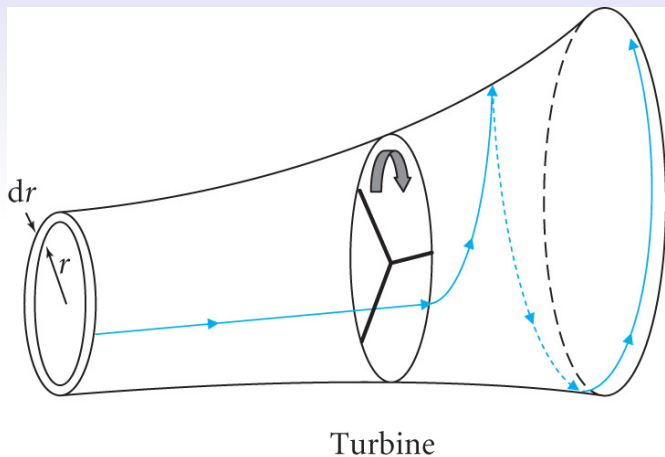# Exchange of angular momentum between air and turbine

Turbine

Figure: From AJ