Solar Energy

PEN : Lectures Week3

The Solar Resource : Nuclear reactions

- Nuclear Reactions in the sun :
  Predominant reaction is the fusion of hydrogen to helium.

$$4\ {}^{1}_{1}H \rightarrow {}^{4}_{2}He + 2e^{+} + 2\nu + 26.2\ MeV$$

  For every 4 hydrogen nuclei, we get 26.2 MeV.

- Mass number = number of neutrons
  Atomic number = number of protons
  Atomic Mass unit : $1.67 \times 10^{-27} Kg$
  Sometimes energy released is given in terms of moles of nuclei (or atoms, molecules ..) rather than units. 1 mole is 6.022×10^{23} units. This is called Avogardro number or Avogadro constant.

The Solar Resource : Nuclear reactions

- The particle ν is a *neutrino*.
- It has zero electric charge, i.e does not interact via the electromagnetic force.
- Also does not carry “colour”. Does not interact via the strong force. Only participates in interactions via the “weak force”
- Discovery ..

The Solar Resource : Nuclear reactions

- In the most common chain of reactions, we have the following steps

$$\begin{aligned} 2\ {}^1_1H &\rightarrow e^+ + \nu + {}^2_1H + 1.2\ MeV \\ {}^2_1H + {}^1_1H &\rightarrow \gamma + {}^3_2He + 5.5\ MeV \end{aligned}$$

γ denotes a photon.

After the above steps are completed twice, we can have

$$2\ {}^3_2He \rightarrow 2{}^1_1H + {}^4_2He + 12.9\ MeV$$

Exercise I : Check that the output of 26.2 MeV for four hydrogen is consistent with the energy released in the individual steps.

- The above accounts for 91% of the sun's energy.
Alternative steps following the first two involve intermediate nuclei ${}^4_2He, {}^7_4Be, {}^7_3Li, {}^8_5B, {}^8_4Be$.
(if interested, see details in Sorensen page 165)

Modelling the Equilibrium state of the Sun

- The hydrogen burning stage is thought to last about $10^{10} yrs$. With an age of around $5 \times 10^9 yrs$ it has reached the half-way stage.
- In the steady state of the sun, the forces due to pressure gradient and gravitation balance each other. This is the condition of hydrostatic equilibrium.

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

- Along with an assumption of ideal gas behaviour

$$P(r) = n(r)k_B T(r)$$

where $n(r) =$ number of free particles per unit volume, and $k_B = 1.38 \times 10^{-23} JK^{-1}$

Modelling the Equilibrium state of the Sun

- For hydrogen : helium fraction $X : (1 - X)$, the number of particles in mass M is

$$2 \times \frac{XM}{m_p} + 3 \times \frac{(1-X)M}{4m_p} = \frac{(3+5X)M}{4m_p}$$

So number of particles per unit mass is

$$\frac{(5X+3)}{4m_p}$$

This gives a relation between $\rho(r)$ and number density $n(r)$

$$\begin{aligned} \rho(r) &= n(r) \times \text{ mass per particle} \\ &= n(r) \frac{4m_p}{(5X+3)} \end{aligned} \quad (1)$$

- Virial theorem relating average kinetic and potential energies.

$$2 \times \frac{3}{2} n(r) k_B T(r) = \frac{GM(r)\rho(r)}{r}$$

- Use (1) in the above to obtain

$$T(r) = \frac{4}{3} \frac{GM(r) m_p}{(5X+3) k_B r}$$

- The average temperature

$$\begin{aligned} \langle T \rangle &= \int \frac{T(r) dM(r)}{M} \\ &= \frac{4}{3} \frac{G m_p}{(5X+3) k_B} \int \frac{M(r) dM(r)}{Mr} \end{aligned}$$

- Approximate

$$\int \frac{M(r) dM(r)}{Mr} \sim \frac{M}{R}$$

where M is the total mass and R is the radius of the sun,

- This leads to a formula

$$T = \frac{4GMM_p}{3(5X+3)k_B R}$$

  for the temperature in terms of total Mass M of sun, Radius R of Sun, $X = 0.7$, mass of proton M_p and Boltzmann ocnstant k_B.
- Composition of Sun assumed $7:3$ of hydrogen and helium by mass.
- For more details on the equations modelling the equilibrium condition of the sun, see Chaper 2A of *B. Sorensen, Renewable energy.*

- Disagreement of neurino fluxes from the sun with the amount predicted from the solar model lead to the discovery of neutrino masses and neutrino oscillations.

Solar Energy received on Earth

- The smooth black body spectrum shown is quite close to what is incident on the upper atmosphere.
- The average flux of solar energy reaching the top of the atmosphere is about $1.36kWm^{-2}$. On the surface of the Earth it is $1.0kWm^{-2}$.
- We say that $I_{AM0} = 1.36kWm^{-2}$ and $I_{AM1.5} = 1.0kWm^{-2}$. *AM* stands for “air mass.”

Planck Spectrum : Energy density

- The energy per unit volume per unit frequency range is given by a function u_ν

$$u_\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

The energy per unit volume in a frequency range $d\nu$ is given by $u_\nu d\nu$.

- The energy per unit volume per unit wavelength range is given by $\tilde{u}_\lambda d\lambda = u_\nu d\nu$.

$$\tilde{u}_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- *Remark* It is important to note that $\tilde{u}(\lambda) \neq u(\nu(\lambda))$.

Wien's displacement Law

- The maximum of u_ν is at ν_{max} given by

$$\frac{h\nu_{max}}{k_B T} = 2.82144$$

- The maximum of $\tilde{u}_\lambda$ is at λ_{max} given by

$$\frac{hc}{k_B T \lambda_{max}} = 4.96511$$

- This relation is called **Wien's Law** . It can be used to show that a black body at Earth's temperature of around $15^\circ C$ radiates in the IR.
- A body at the $3K$ radiates in the microwave region. THe cosmic microwave background.
- Note that $\lambda_{max}\nu_{max} \neq c$. This is because $\tilde{u}(\lambda) = \frac{c}{\lambda^2} u(\nu(\lambda)) \neq u(\nu(\lambda))$

Planck Spectrum : Flux

- The Flux F per unit frequency range i.e power per unit area per unit frequency range, is $u_\nu c$.
- This follows because

$$F.d\nu.Adt = u_\nu d\nu dV$$

  and $dV = Acdt$.
- Figure in class.

$$F = \frac{8\pi h}{c^2} \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

Planck Spectrum : Low frequency limit

- At low fequencies, use $e^x = 1 + x$.
- Hence derive $u_\nu = \frac{8\pi k_B T \nu^2}{c^3}$ or $u_\lambda = \frac{8\pi k_B T}{\lambda^4}$.
- Rayleigh-Jeans Law.

Solar Energy Incident on Earth

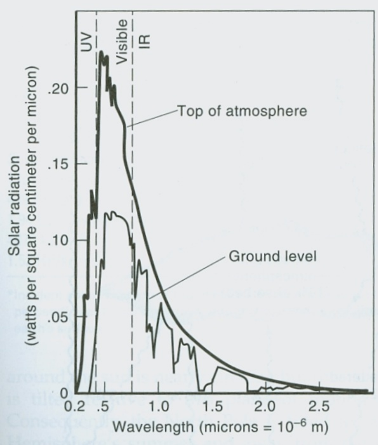

FIGURE 6.2
Spectrum of solar radiation reaching the earth at the top of the atmosphere and at ground level. (The minima in the ground level spectrum are a result of the absorption by water vapor, CO_2, O_2, N_2, and ozone [O_3].) About 40% of the solar radiation is in the visible region.

Figure: solar resource

- Figure (from HK) gives the distribution of energy across the spectrum, in the upper atmosphere, and at the ground level.

- Solar radiation reaching the Earth is direct (focusable by mirrors) and diffuse (unfocusable).
- Diffuse light reaches the Earth's surface after being scattered by molecules in the air and the clouds. Amount depends on cloudiness. Typical yearly average is 30%

Solar Energy : General types of devices and systems

- Solar Hot Water and Heating systems : Active and Passive. Schematic from HK

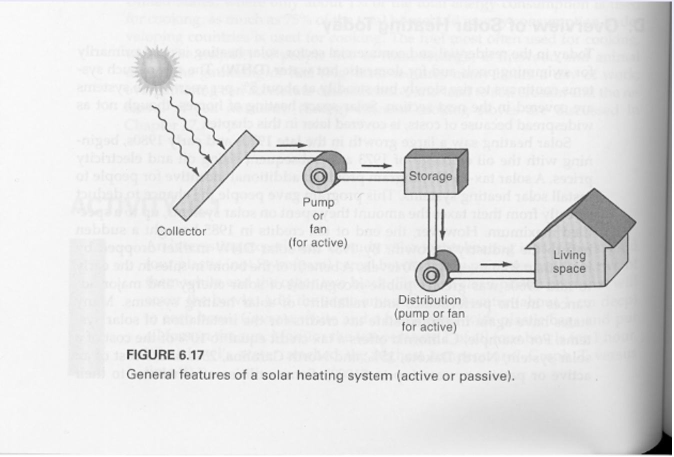

FIGURE 6.17
General features of a solar heating system (active or passive).

Figure: Solar water

Earth’s axis and orbit

- Device has to be south facing. Optimal angles requires some celestial trigonometry. Detailed treatment can be found in Sorensen 3.1.1

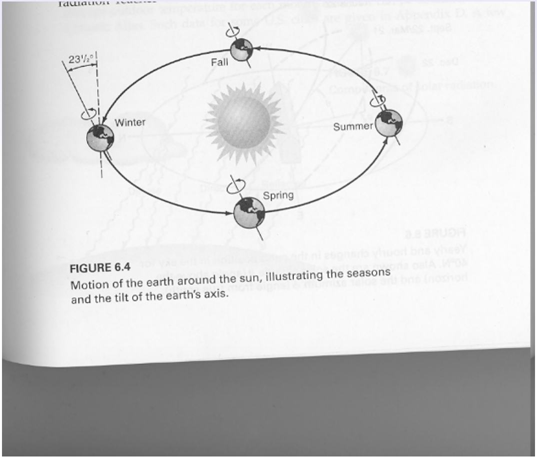

FIGURE 6.4
Motion of the earth around the sun, illustrating the seasons and the tilt of the earth’s axis.

Figure: Tilt of Earth’s axis

Trajectory of sun in the Sky

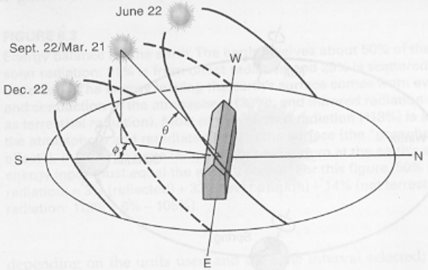

FIGURE 6.6
Yearly and hourly changes in the sun's position in the sky for 40°N. Also shown are the solar altitude θ (angle above the horizon) and the solar azimuth ϕ (angle from true south).

Figure: trajectory of sun

The angle between the sun and the vertical, denoted by θ can be computed in terms of the solar declination δ, the latitude ϕ and the hour angle *h*, using

$$\cos\theta = \sin\phi\sin\delta + \cos\phi\cos\delta\cos h \tag{2}$$

The declination is fixed by the time of the year. Consider at the chosen time of the year, the latitude at which the sun is highest at noon. This is δ. During summer solstice (June 22), it is 23.5°. At the equinoxes, it is zero. During winter solstice it is $-23.5°$. See the picture showing tilt of the Earth's axis.
For a chosen point of interest on the Earth, the hour angle varies with time of the day. At noon it is 0.0 degree. It changes from $-\pi$ to π over 24 hours. It is also the difference in longitude between the place where it is noon to the place of interest.

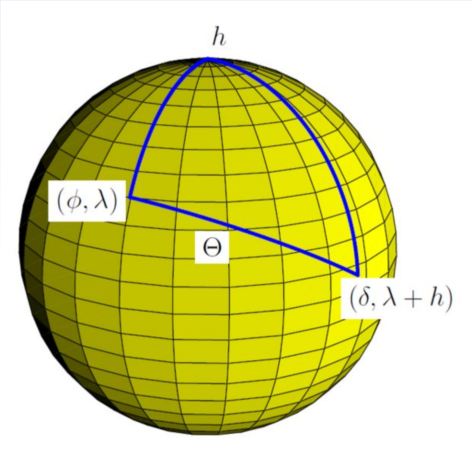

Figure: Angles and insolation

The formula above is derived using the spherical law of cosines applied to the figure below.

$$\cos c = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(C)$$

with

$$\begin{aligned} C &= h \\ c &= \theta \\ a &= \frac{\pi}{2} - \phi \\ b &= \frac{\pi}{2} - \delta \end{aligned}$$

Solar Energy : General types of devices and systems

- Solar Thermal power plants
  - **Ocean Thermal Energy Conversion (OTEC)**
    Oceans absorb solar radiation and heat up. Top hundred metres approx. $20 - 25^{\circ}C$ higher than deep below. Drive a heat engine which exploits this temperature difference, drives a turbine and generates electricity.
  - **Solar Driven Stirling Engines**
    Solar collecting dishes on an area $11km \times 11km$. The dishes focus sunlight to heat up hydrogen. The heated gas does work, cools down and then is heated up again. More details on this in Box 6.4 of AJ.
    See also http://www.stirlingenergy.com/
  - **Solar Chimneys**
    Warm air is produced under a large area of glass and rises up a tall chimney, thereby running wind turbines which generate electricity. Conversion efficiency is approx. 2%, but with cheap land and plenty of sunshine, cost per kWh makes financial sense.

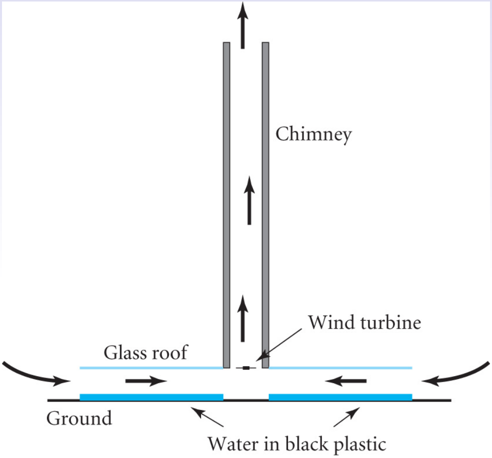

Figure: Solar Chimney

Direct conversion Light to Electricity

- Photovoltaic systems. Realistic systems have 15% efficiency.
- Direct conversion desirable, since efficiencies multiply in a multi-stage process.