CAPACITANCE

Definition of capacitance

Recall: For a point charge or a charged sphere $V = \dfrac{Q}{4\pi\varepsilon_o r}$

In general, POTENTIAL $\propto$ CHARGE for any size or shape of conductor.

Definition: The constant of proportionality between V and Q is called CAPACITANCE, C:

$$C = \frac{Q}{V}$$

Units of capacitance: $C \equiv \dfrac{\text{Coulombs}}{\text{Volts}} \equiv CV^{-1}$

Definition: $1 \text{ Farad (F)} \equiv 1\ CV^{-1}$

Capacitance is a measure of the ability of a conductor or a system of conductors to store charge (and hence to store energy).

Recall: For a charged conductor $U_{tot} = \dfrac{1}{2}QV$

$$\Rightarrow \quad U_{tot} = \frac{1}{2}CV^2 \qquad \text{or} \qquad U_{tot} = \frac{1}{2}\frac{Q^2}{C}$$

Relationship between capacitance and energy

Alternative units for ε_o:

$$E = \frac{Q}{4\pi\varepsilon_o r^2} \equiv \frac{\text{Coulombs}}{\varepsilon_o \text{metres}^2} \equiv \frac{\text{Volts}}{\text{metres}}$$

$$\Rightarrow \quad \varepsilon_o \equiv \frac{\text{Coulombs}}{(\text{Volts})(\text{metres})} \equiv \frac{\text{Farads}}{\text{metres}}$$

$\Rightarrow \quad \varepsilon_o \equiv Fm^{-1}$ These are the units in which ε_o is usually quoted.

Procedure for finding capacitance

1. Imagine a charge Q on the conductor (if it's a single conductor) or charges of ±Q (for a pair of conductors)

2. Find $\overline{\mathbf{E}}$ (e.g., use Gauss's Law)

3. Find the potential (difference) using $|\Delta V| = \left|\int_a^b \overline{\mathbf{E}} \cdot d\overline{\mathbf{L}}\right|$

   (never mind the sign).

4. Put C = Q/V [Q always cancels out]

Note:

1. C depends only on the geometry - the size and shape of the conductors and the distance between them.

2. C is independent of Q [because $V \propto Q$]

Examples:

1. Capacitance of an isolated sphere
2. Parallel plate capacitor
3. Capacitance of two concentric spheres
4. Capacitance per unit length of a co-axial cable

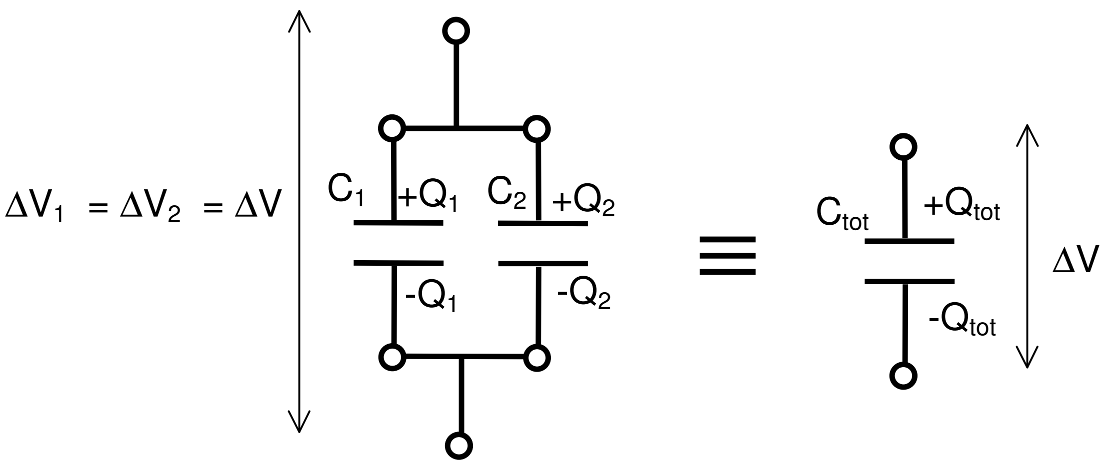
See lecture notes

Capacitor

An electrical component designed to have a particular value of C.

Capacitors in parallel

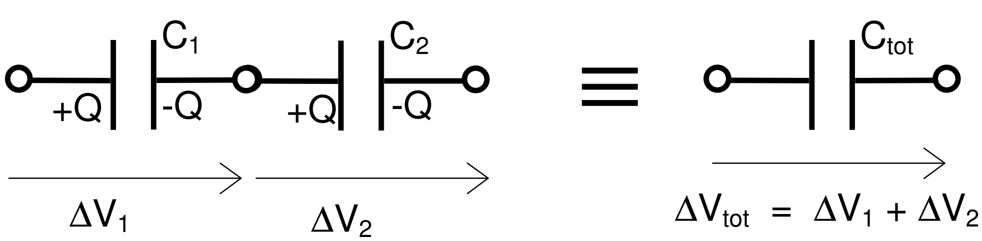
ΔV is the same for both: $\Delta V_1 = \Delta V_1 = \Delta V_{tot}$



$Q_{tot} = Q_1 + Q_2$ $\Delta V_{tot} = \Delta V$

Therefore $C_{tot} = \dfrac{Q_1}{\Delta V} + \dfrac{Q_2}{\Delta V} \Rightarrow \boxed{C_{tot} = C_1 + C_2}$

Capacitors in series



In this case Q is the same for both and ΔV is different.

$$C_1 = \frac{Q}{\Delta V_1} \qquad C_2 = \frac{Q}{\Delta V_2} \qquad C_{tot} = \frac{Q_{tot}}{\Delta V_{tot}} = \frac{Q}{\Delta V_1 + \Delta V_2}$$

$$\frac{1}{C_{tot}} = \frac{\Delta V_1}{Q} + \frac{\Delta V2}{Q} \Rightarrow \boxed{\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Dielectrics and polarisation

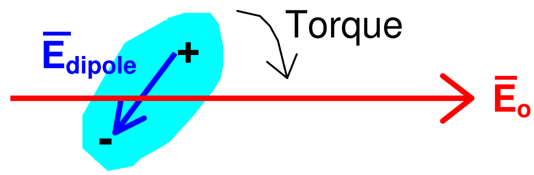
Until now, we have assumed that the space between charges, conductors, etc. is empty (a vacuum). What if it's filled with some insulating material?

Recall: A DIELECTRIC (insulator) is electrically neutral.

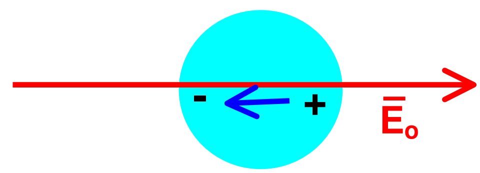
But it contains many +ve and -ve charges in its atoms or molecules. Because it's an insulator, the charges can't move around.

BUT: some molecules have a natural DIPOLE MOMENT

$\Rightarrow$ when placed in an external electric field $\overline{\mathbf{E}}_{\mathbf{o}}$, the dipoles tend to align with it.

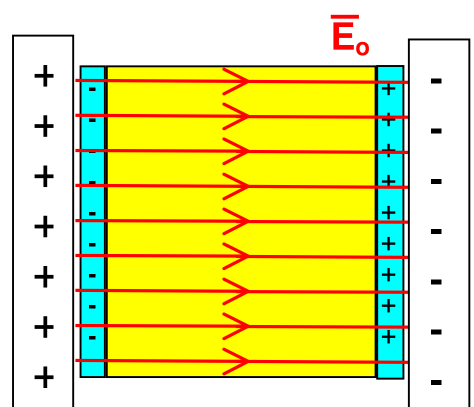


Even if no INTRINSIC dipoles exist, they can be INDUCED by an applied field $\overline{\mathbf{E}}_{\mathbf{o}}$, producing the same effect.



Consider a parallel plate capacitor, with positive and negative charges on the plates. This creates an electric field $\overline{\mathbf{E}}_{\mathbf{o}}$ between the plates.

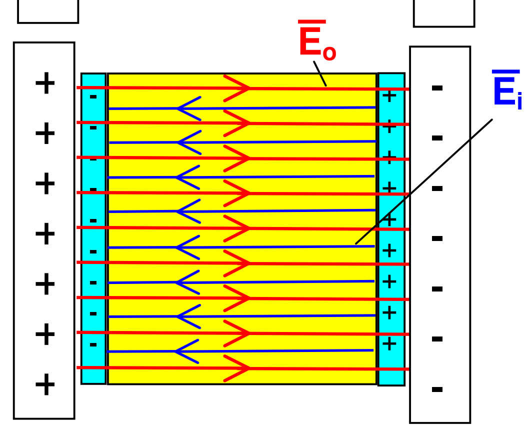
If there is a dielectric between the plates, then due to dipole alignment, or (POLARISATION) the distributions of +ve and -ve charge do not overlap exactly:



Excess +ve charge on the right.

Excess -ve charge on the left.

The induced (POLARISED) field, $\overline{\mathbf{E}}_{\mathbf{i}}$, tends to OPPOSE $\overline{\mathbf{E}}_{\mathbf{o}}$.



The resultant field is LESS than the applied field:

$$E_{tot} = E_o - E_i$$

Dielectric constant

$E_{tot} = E_o - E_i$ Let $E_{tot} = \frac{1}{K} E_o$ $(K \geq 1)$

K (sometimes written as the greek letter κ - kappa) is the DIELECTRIC CONSTANT or RELATIVE PERMITTIVITY of the material.

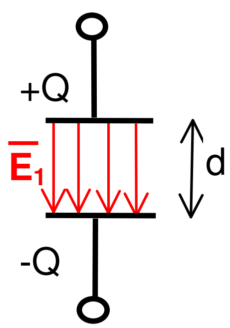
Let σ_o = charge density on capacitor plates: $E_o = \sigma_o/\varepsilon_o$
Let σ_i = induced charge density on surfaces of dielectric: $E_i = \sigma_i/\varepsilon_o$

Therefore $\sigma_i = \varepsilon_o E_i = \varepsilon_o [E_o - E_{tot}] = \varepsilon_o E_o \left[1 - \frac{1}{K}\right]$

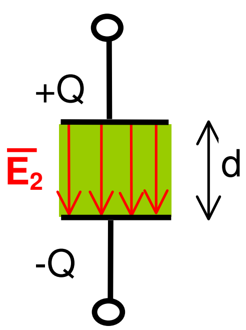
So the induced surface charge density is $\sigma_i = \sigma_o \left[1 - \frac{1}{K}\right]$

The effect of dielectric constant on capacitance

Consider the case of a parallel plate capacitor.



No dielectric: $C_1 = \frac{Q}{\Delta V_1} = \frac{Q}{E_1 d} = \frac{\varepsilon_o A}{d}$



With dielectric: $C_2 = \frac{Q}{\Delta V_2} = \frac{Q}{E_2 d} = \frac{KQ}{E_1 d} = \frac{K\varepsilon_o A}{d}$

Capacitance with dielectric = K(Capacitance without dielectric)

So, capacitance (i.e., the ability to store charge and hence energy) is increased by the use of a dielectric.

Typical values of dielectic constant K

Air	1.00059	(not much different from a vacuum)
Polythene	2.3	
Glass	5 - 10	
Germanium	16	
Water	80	
A perfect conductor	?	

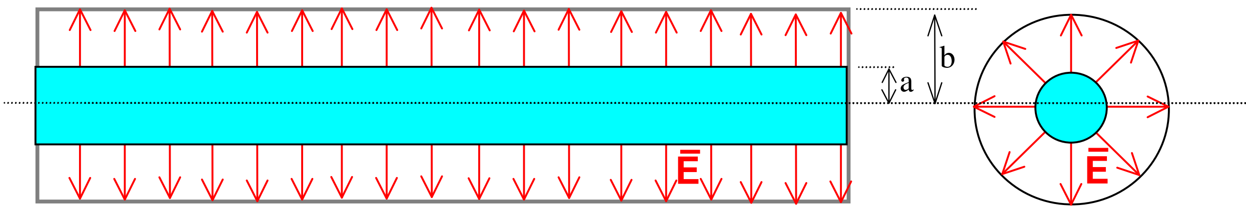
Gauss's Law in dielectrics

Just replace ε_o with $K\varepsilon_o$: $\Phi = \oint \overline{\mathbf{E}} \cdot d\overline{\mathbf{A}} = \dfrac{Q_{enclosed}}{K\varepsilon_o}$

Dielectric breakdown

If E > some limit, the BREAKDOWN FIELD STRENGTH of the material, then the bonds between the electrons and the atoms are broken and the material becomes a CONDUCTOR ($\rightarrow$ short circuit in a cable or capacitor).

Capacitance per unit length of a pair of co-axial cylinders (e.g., a co-axial cable)



Step 1: Let charge per unit length be $+\lambda$ on inner conductor
$-\lambda$ on outer conductor

Step 2: Find $\bar{\mathbf{E}}$:

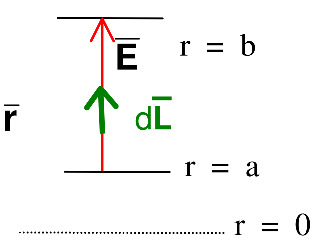
Field pattern: Field lines go from +ve charges on inner conductor to -ve charges on outer conductor

E = 0 for r < a and r > b (A Gaussian cylinder encloses no charge)

Apply Gauss's Law $\rightarrow$ $$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

(similar to a previous example)

Step 3: Find ΔV: $|\Delta V| = \left| \int \bar{\mathbf{E}} \cdot d\bar{\mathbf{L}} \right|$



Path: Integrate along a field line $\rightarrow$ $d\bar{\mathbf{L}} = d\bar{\mathbf{r}}$

and $d\bar{\mathbf{L}}$ is parallel to $\bar{\mathbf{E}}$ so $\bar{\mathbf{E}} \cdot d\bar{\mathbf{L}} = Edr$

$$|\Delta V| = \left| \int_a^b \frac{\lambda}{2\pi\varepsilon_o r} dr \right| = \left| \frac{\lambda}{2\pi\varepsilon_o} \ln\left(\frac{b}{a}\right) \right|$$

We take ΔV to be positive when finding capacitance.

Step 4: Put C = Q/V $\rightarrow$ $C = \dfrac{2\pi\varepsilon_o}{\ln\left(\dfrac{b}{a}\right)}$ (Capacitance/unit length)