ELECTRIC AND MAGNETIC FIELDS
ANSWERS TO WEEK 1 ASSIGNMENT

15 **Q1** $\bar{A} = -8\hat{i} + 2\hat{j} + 6\hat{k}$ and $\bar{B} = 3\hat{i} - 3\hat{j} + 3\hat{k}$

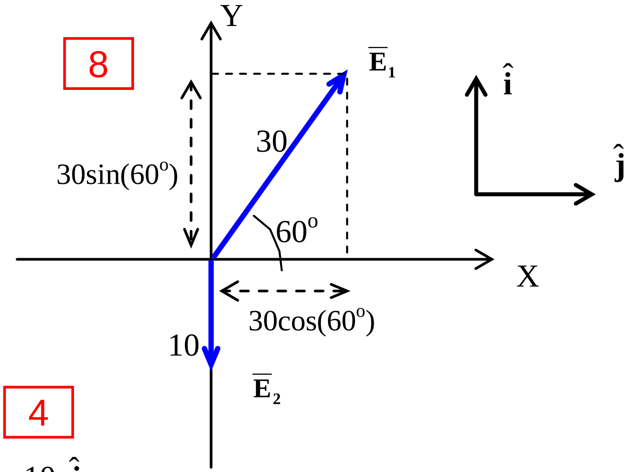
$\bar{A} + \bar{B} = (-8+3)\hat{i} + (2-3)\hat{j} + (6+3)\hat{k} = -5\hat{i} - 1\hat{j} + 9\hat{k}$ 5

$\bar{A} - \bar{B} = (-8-3)\hat{i} + (2+3)\hat{j} + (6-3)\hat{k} = -11\hat{i} - 5\hat{j} + 3\hat{k}$ 5

$4\bar{A} - 3\bar{B} = (-32-9)\hat{i} + (-8+9)\hat{j} + (24-9)\hat{k} = -41\hat{i} + \hat{j} + 15\hat{k}$ 5

20 **Q2**

(i) Diagram: 8



(ii) Express vectors in terms of orthogonal unit vectors:

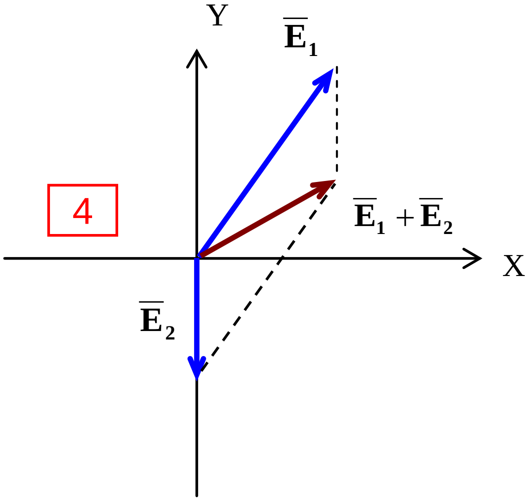
$\bar{\mathbf{E}}_1 = 30\text{Cos}(60^o)\ \hat{\mathbf{i}} + 30\sin(60^o)\ \hat{\mathbf{j}}$

So $\bar{\mathbf{E}}_1 = 15\ \hat{\mathbf{i}} + 26.0\ \hat{\mathbf{j}}$ 4

$\bar{\mathbf{E}}_2$ only has a Y-component: $\bar{\mathbf{E}}_2 = -10\ \hat{\mathbf{j}}$

Adding $\bar{\mathbf{E}}_1$ and $\bar{\mathbf{E}}_2$, we get $\bar{\mathbf{E}}_1 + \bar{\mathbf{E}}_2 = 15\ \hat{\mathbf{i}} + 16.0\ \hat{\mathbf{j}}$ 4

(iii) Illustrate the resultant: 4



15 **Q3** Find a vector of magnitude 27 which is perpendicular to $\overline{A} = 6\hat{i} + 6\hat{j} - 9\hat{k}$

~ 10 for the method

~ 5 for getting the exact answer

If we divide any vector by its magnitude, we get a UNIT vector with the same direction. Here, we want the magnitude to be 27, so we simply multiply the result by 27. To reverse the direction, we then change the signs of the three components.

Magnitude of $\overline{\mathbf{A}}$ is $A = (6^2 + 6^2 + 9^2)^{1/2} = 12.37$

$\Rightarrow$ Unit vector in direction of $\overline{\mathbf{A}}$ is

$$\hat{a} = \frac{6}{12.37}\hat{i} + \frac{6}{12.37}\hat{j} - \frac{9}{12.37}\hat{k}$$

$\Rightarrow$ Vector of magnitude 27, opposite to $\overline{\mathbf{A}}$ is

$$-27\hat{a} = -13.1\hat{i} - 13.1\hat{j} + 19.6\hat{k}$$

15 **Q4** $\overline{\mathbf{A}} = -3\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ and $\overline{B} = 2\hat{i} - 3\hat{j} + 5\hat{k}$.

8

(i) Dot product: $\overline{\mathbf{A}} \cdot \overline{\mathbf{B}} = (-3)(2) + (-8)(-3) + (7)(5) = 53$

(ii) Angle between A and B: By definition, $\overline{\mathbf{A}} \cdot \overline{\mathbf{B}} = AB\cos\theta$

So $(-3^2 + -8^2 + 7^2)^{1/2}(2^2 + -3^2 + 5^2)^{1/2}\text{Cos}\theta = 53$

Therefore $\text{Cos}\theta = 53/68.1 = 0.78 \Rightarrow \theta = 38.9^o$

7

15 **Q5**

$$\overline{P} \times \overline{E} = (3\hat{i} - 5\hat{j}) \times (2\hat{i} - 4\hat{j})$$

$$= (3\hat{i} \times 2\hat{i}) + (3\hat{i} \times -4\hat{j}) + (-5\hat{j} \times 2\hat{i}) + (-5\hat{j} \times -4\hat{j})$$

$$= 6(\hat{i} \times \hat{i}) - 12(\hat{i} \times \hat{j}) - 10(\hat{j} \times \hat{i}) + 20(\hat{j} \times \hat{j})$$

Now, $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0 \qquad \hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{i} = -\hat{k}$

Therefore $\overline{P} \times \overline{E} = -2\hat{k}$

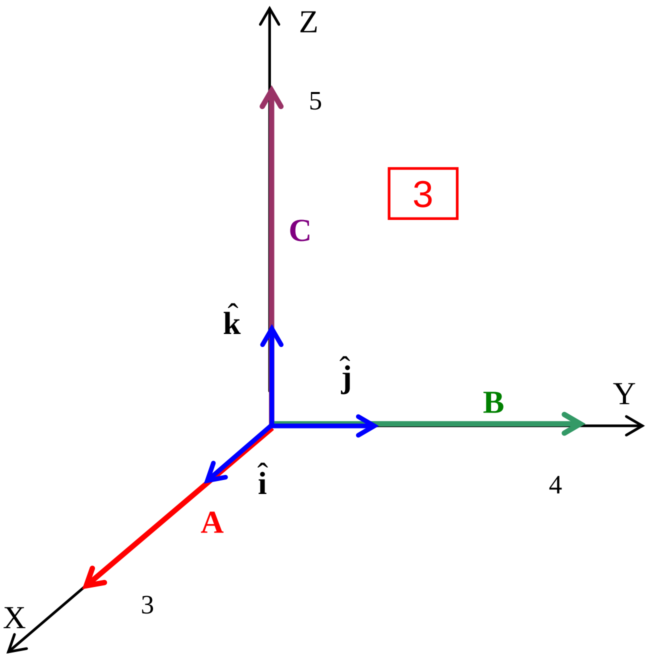
~ 10 for method

~ 5 for getting the exact answer

Zero marks for using the determinant method

15 **Q6**

(i) Diagram showing the x, y and z axes, the orthogonal unit vectors, and the vectors **A**, **B** and **C**.



(ii)

$\overline{A}\times\overline{B}=20(\hat{i}\times\hat{j})=20\hat{k}$ 3

$\overline{A}\times\overline{C}=15(\hat{i}\times\hat{k})=-15\hat{j}$ 3

$\overline{C}\times\overline{B}=12(\hat{k}\times\hat{j})=-12\hat{i}$ 3

$\overline{\mathbf{A}}\cdot\overline{\mathbf{B}} = \overline{\mathbf{A}}\cdot\overline{\mathbf{C}} = \overline{\mathbf{C}}\cdot\overline{\mathbf{B}}=0$ 3
(they are all perpendicular to each other)

5 **Q7** $\overline{\mathbf{A}} = -2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Vector, $\overline{\mathbf{B}}$, has magnitude $90^{1/2}$, lies in the first quadrant of the x-y plane, and has direction perpendicular to $\overline{\mathbf{A}}$.

Let the required vector be $\overline{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$ 1

$\overline{\mathbf{B}}$ is in the x-y plane $\Rightarrow$ $B_z = 0.$ 1

$\overline{\mathbf{A}}$ and $\overline{\mathbf{B}}$ are perpendicular $\Rightarrow$ $\overline{\mathbf{A}}\cdot\overline{\mathbf{B}} = 0$

$\Rightarrow$ $-2B_x + 6B_y = 0$

$\Rightarrow$ $B_x = 3B_y$ 1

$\overline{\mathbf{B}}$ has magnitude $40^{1/2}$ $\Rightarrow$ $B_x^2 + B_y^2 = 90$ 1

$\Rightarrow$ $10\,B_y^2 = 90$

$\Rightarrow$ $B_y = \pm 3$ and $B_x = \pm 9$

$\overline{\mathbf{B}}$ is in the first quadrant, so B_x and B_y are positive $\Rightarrow$
$\overline{\mathbf{B}} = 9\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$. 1

Mark on method using this as a guide. Several ways of getting answer, plus this is a diificult question so expect most to only get some of the way there.