Queen Mary
University of London

BSc/MSci EXAMINATION

PHY-121 Mathematical Techniques I

Time Allowed: 2 hours 30 minutes

Date: 20^{th} May, 2010

Time: 14:30 - 17:00

Instructions: **Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 50 marks, each question in section B carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. Course work comprises 20% of the final mark.**

You may wish to use the following information:-

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Important Note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure that you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Examiners: Dr. A. Bevan, Dr. A. Martin

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SECTION A. Attempt answers to all questions.

A1

(a) Differentiate the function $y = \cos(x)$ with respect to x.

(b) Differentiate the function $y = e^{ax}$ with respect to x.

(c) Differentiate the function $y = x^2(2x + 1)$ with respect to x.

(d) Differentiate the function $y = (2x + 1)/(x^2 - 1)$ with respect to x.

(e) Differentiate the function $y = e^{\sin(x)}$ with respect to x.

(f) Determine all first and second derivatives of the function $z = xy - x^2e^y - 3$, where x and y are independent variables.

[10]

A2

(a) Evaluate the following integral $\int \cos(x)dx$.

(b) Evaluate the following integral $\int 3x + e^x dx$.

(c) Evaluate the following integral $\int x \ln x - xdx$.

(d) Evaluate the following integral

$$\int \frac{2x}{x^2 + 1} dx.$$

(e) Evaluate the following integral

$$\int \frac{1}{(x - 1)(2x + 1)} dx.$$

[10]

A3

(a) Expand $\sum_{k=1}^{3} kx^k$.

(b) Write down the summation equation for a geometric progression with n terms, and a common ratio between the successive terms of r.

(c) Write down the first four terms in the equation for a Maclaurin series expansion.

(d) Write down the first four terms in the equation for a Binomial series expansion.

(e) State L'Hôpital's rule.

[10]

A4

(a) Compute i^3.

(b) Compute $(1+i)+(1-i)$, and express your solution in the form $a+ib$.

(c) Compute $(1+i)(1-i)$, and express your solution in the form $a+ib$.

(d) Compute $(1+i)/(1-i)$, and express your solution in the form $a+ib$.

(e) Express $(1+i)$ in the form $re^{i\theta}$.

(f) Compute all the cubed roots of -1, and plot these on an Argand diagram.

[10]

A5

(a) Write down the form of a Fourier series expansion in terms of a sum of sine and cosine parts. In doing so, write equations for the associated coefficients A_0, A_n, and B_n. Also state what the value obtained for A_0 corresponds to.

(b) Write down the equation for a delta function centred about $x = x_0$, and note the normalization condition for such a function.

(c) Simplify $\int_{-\infty}^{\infty} f(x)\delta(x-3)dx$.

[10]

SECTION B. Answer two of the four questions in this section.

B1

a) Compute the first and second derivatives of the function $y = \sin(x)/x$, hence determine the radius of curvature at the point $x = \pi$. **[10]**

b) Determine the location of the centre of curvature corresponding to the point on the curve given by $x = \pi$. **[5]**

c) Determine the position and nature of any stationary points of the function $z = e^x + e^{-x} - y^2$. **[10]**

[TOTAL FOR B1 = 25]

B2

a) Determine the centroid position $\overline{x}$ for the lamina given by the curve $y = xe^x$ bounded by the x-axis, and the lines $x = 0$ and $x = 1$. **[10]**

b) If the lamina in part (a) is rotated about the x-axis it forms a volume in a three-dimensional space (x, y, z). Compute the centroid position $(\overline{x}, \overline{y}, \overline{z})$ of that volume, using symmetry where appropriate. **[15]**

[TOTAL FOR B2 = 25]

B3

a) Find the limiting value of the following when $x \to \infty$

$$\frac{x^5 + 7x^2 + 1}{3x^5 + 2}.$$

[5]

b) Using the first four terms of a binomial series expansion, estimate $\sqrt[4]{0.8}$. **[5]**

c) Compute the first four non-zero terms of the series expansion for the function $x\cosh(x)$ when expanding about $x = 0$. **[5]**

d) Given two complex numbers $A_1 = ae^{i\theta}$, and $A_2 = be^{i\phi}$, for the sum $A = A_1 + A_2$ compute $|A|^2 = AA^*$. **[5]**

e) Simplify your result from B3 part (d) for the special case when $\theta = \phi$. **[5]**

[TOTAL FOR B3 = 25]

B4

a) Consider the function determined by $y = h$ when $0 \leq x + nT \leq 1$, $y = 0$ elsewhere, where $T = 2$ is the periodicity of the function. Determine the Fourier series expansion for y.

[10]

b) Sketch the function in B4 part (a), and overlay the distributions corresponding to the sum of the constant and first harmonic of y on the same plot. **[5]**

c) Compute the Fourier transform of the function given by $y(x) = 1 - x$ for $0 \leq x \leq 1$, and $y(x) = 0$ elsewhere. **[10]**

[TOTAL FOR B4 = 25]