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SCHOOL OF PHYSICS AND ASTRONOMY

Structure and Properties of Functional Materials

Homework Set 6

Due Wednesday, 13 March, 2013 by 4 p.m.

Problem 1: Terms and definitions (8 marks)

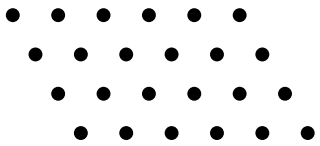
Explain the following terms or concepts, giving an example of their significance in condensed matter physics:

(a) Brillouin zone (4)

(b) Semiconductor (4)

Problem 2: A hexagonal metal in two dimensions (17 marks)

Consider a divalent, two-dimensional, hexagonal metal:



(a) Sketch the first, second, and third Brillouin zones. (6)

(b) Sketch the Fermi surface, and shade the occupied states, in each of the following cases: (6)

i. A free electron gas with no band gap;

ii. A nearly-free electron gas with a small band gap;

iii. A band gap big enough that this system becomes an insulator.

(c) What is the maximum bandgap (at $\mathbf{k} = \frac{1}{2}\mathbf{a}^*$, which is labelled M in hexagonal systems) such that this system remains a metal? (5)

Problem 3: Periodicity in reciprocal space (5 marks)

Show that the wavefunctions for states whose wavevectors differ by a reciprocal lattice vector are identical; that is, that

$$\psi_{\mathbf{k}+\mathbf{G}}(x) = \psi_{\mathbf{k}}(x).$$

Hence justify our claim in class that electronic dispersion curves are periodic in reciprocal space.

Data:

Electronic charge	$e = 1.6022 \times 10^{-19}\,\mathrm{C}$
Planck constant	$h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k_\mathrm{B} = 1.3807 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron mass	$m = 9.109 \times 10^{-31}\,\mathrm{kg}$
Avogadro number	$N_\mathrm{A} = 6.022 \times 10^{23}\,\mathrm{mol^{-1}}$