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# Structure and Properties of Functional Materials

## Homework Set 7

*Due Wednesday, 20 March, 2013 by 4 p.m.*

**Problem 1: Terms and definitions** (8 marks)

Explain the following terms or concepts, giving an example of their significance in condensed matter physics:

(a) Effective mass (4)

(b) Intrinsic conduction (4)

**Problem 2: Effective mass** (13 marks)

Consider a one-dimensional solid with lattice spacing $a$. Suppose that it contains a band with the dispersion relationship

$$E(k) = E_0 - 2I\cos(ka)$$

for some constants $E_0$ and $I$. This band contains only one electron, which is initially at rest at $x = 0$.

At time $t = 0$ an electric field $\mathcal{E}$ is turned on in the positive $x$ direction. Sketch the electron's position $x$, velocity $v$, and effective mass $m^*$ as a function of time $t$. Comment on the relationship between these quantities.

**Problem 3: Excitons** (7 marks)

Electrons and holes can interact via the Coulomb attraction to form pairs known as *excitons*. Just as we saw in class for dopants, these can be described using the Bohr model of hydrogen. This time we must replace the electron mass by a *reduced effective mass* $\mu^*$ given by

$$\frac{1}{\mu^*} = \frac{1}{m_c^*} + \frac{1}{m_{hh}^*},$$

and just as in class we must also allow for the dielectric constant $\epsilon$ of the material.

Consider an exciton in the semiconductor indium nitride (InN; $m_c^* = 0.11m_e$; $m_{hh}^* = 1.63m_e$; $\epsilon = 8.4$). Calculate the binding energy of the exciton and its effective Bohr radius. Comment on the magnitudes of these quantities.

**Problem 4: Intrinsic and extrinsic conduction** (12 marks)

(a) Calculate the density $n$ of electrons in the valence band of intrinsic Ge (take $m_c^* = 0.22m_e$; $m_{hh}^* = 0.33m_e$; $E_g = 0.661\,\text{eV}$) at $T = 300\,\text{K}$. Hence estimate the maximum impurity concentration tolerable if this material is to behave as an intrinsic semiconductor at room temperature. Is this level of purity practically possible? (5)

(b) Suppose instead that we have a sample doped with P donors at a concentration of 1 part in $10^6$. What is the density of electrons in the valence band now? What is the density of holes in the conduction band? (Ge crystallises in the diamond structure, with eight atoms in the cubic unit cell of side length $a = 5.658\,\text{Å}$.) (7)

**Data:**

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|---|---|
| Electronic charge | $e = 1.6022 \times 10^{-19}\,\mathrm{C}$ |
| Planck constant | $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$ |
| | $\hbar = h/2\pi = 1.055 \times 10^{-34}\,\mathrm{J\,s}$ |
| Boltzmann constant | $k_\mathrm{B} = 1.3807 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ |
| Electron mass | $m = 9.109 \times 10^{-31}\,\mathrm{kg}$ |
| Avogadro number | $N_\mathrm{A} = 6.022 \times 10^{23}\,\mathrm{mol^{-1}}$ |