Queen Mary
University of London

BSc Examination by course unit.

Friday 25th May 2012 10:00 – 12:30

PHY214 Thermal & Kinetic Physics Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 50 marks, each question in section B carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. Course work comprises 20% of the final mark

CALCULATORS ARE PERMITTED IN THIS EXAMINATION. PLEASE STATE ON YOUR ANSWER BOOK THE NAME AND TYPE OF MACHINE USED.

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK THAT IS NOT TO BE ASSESSED.

IMPORTANT NOTE: THE *ACADEMIC REGULATIONS* STATE THAT POSSESSION OF UNAUTHORISED MATERIAL AT ANY TIME WHEN A STUDENT IS UNDER EXAMINATION CONDITIONS IS AN ASSESSMENT OFFENCE AND CAN LEAD TO EXPULSION FROM QMUL.

PLEASE CHECK NOW TO ENSURE YOU DO NOT HAVE ANY NOTES, MOBILE PHONES OR UNATHORISED ELECTRONIC DEVICES ON YOUR PERSON. IF YOU HAVE ANY THEN PLEASE RAISE YOUR HAND AND GIVE THEM TO AN INVIGILATOR IMMEDIATELY. PLEASE BE AWARE THAT IF YOU ARE FOUND TO HAVE HIDDEN UNAUTHORISED MATERIAL ELSEWHERE, INCLUDING TOILETS AND CLOAKROOMS IT WILL BE TREATED AS BEING FOUND IN YOUR POSSESSION. UNAUTHORISED MATERIAL FOUND ON YOUR MOBILE PHONE OR OTHER ELECTRONIC DEVICE WILL BE CONSIDERED THE SAME AS BEING IN POSSESSION OF PAPER NOTES. MOBILE PHONES CAUSING A DISRUPTION IS ALSO AN ASSESSMENT OFFENCE.

EXAM PAPERS CANNOT BE REMOVED FROM THE EXAM ROOM

Examiners: K.J.Donovan, T.J.S.Dennis

© Queen Mary, University of London, 2012

SECTION A. Answers all questions in Section A

Question A1

Write down the Boltzmann equation that describes the relationship between the entropy of a macrostate, S, and the number of distinct microstates, Ω, that can make the macrostate in question.

[5 marks]

Question A2

During an adiabatic process on an ideal gas, what is the relationship between the initial pressure and temperature, P_i and T_i , and the final pressure and temperature, P_f and T_f? Explain any terms used.

[5 marks]

Question A3

What is the internal energy of a gas of N rigid diatomic molecules in equilibrium at temperature, T, in terms of the temperature? What is the average kinetic energy of one of the molecules in terms of T?

[5 marks]

Question A4

i) Write down the thermodynamic identity for a P-V system explaining all symbols used.

ii) Write down the thermodynamic identity for a paramagnetic system explaining all symbols used.

[5 marks]

Question A5

i) Give ***one*** example of an intensive property of a P-V system.

ii) Give ***one*** example of an extensive property of a P-V system.

iii) Is the entropy, S, of a system an extensive or an intensive property?

iv) Is the total magnetisation, $\mathcal{M}$, of a paramagnetic system an extensive or an intensive property?

[4 marks]

Question A6

A refrigerator operating between a hot and cold reservoir delivers heat, Q_2, from the cold reservoir to the working system whilst giving up heat, Q_1, to the hot reservoir. Write down the refrigerator efficiency, η_R, in terms of these heat flows.

[5 marks]

Question A7

A Carnot heat pump operates between a hot reservoir at temperature, T_1, and a cold reservoir at temperature, $T_{2,}$ Write down the reversible heat pump efficiency, η_{HP} , in terms of the reservoir temperatures.

[5 marks]

Question A8

Write down a definition of the thermal expansion coefficient, β, of a P-V system in terms of thermodynamic variables and a partial differential. Make it clear which thermodynamic variable is being held constant.

[5 marks]

Question A9

Express the heat capacity of a gas at constant pressure, C_P, in terms of a partial differential ***using state variables***. Define all of the symbols used.

[5 marks]

Question A10

An ideal gas is expanded in an isothermal process at temperature, T, from volume, V, to a volume, $2V$. Write down

i) the work done ***including sign*** and state whether this is work ***done on or by the gas;***

ii) the heat evolved during the process ***including sign*** and state whether this heat ***is absorbed by the gas or environment.***

[6 marks]

SECTION B. Attempt two of the four questions in this section

Question B1

a)

i) Demonstrate the truth of the cyclic relation for partial differentials using a P-V-T system and the equation of state of an ideal gas.

ii) The equation of state of an elastic band under tension, $\mathcal{F}$, at temperature, *T,* is given by;

$$\mathcal{F} = aT\left[\frac{L}{L_0} - \left(\frac{L_0}{L}\right)^2\right].$$

Where *L* is its length while *a* and L_0 are constants.

Demonstrate that the thermal expansion coefficient, $\alpha = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_{\mathcal{F}}$, is given by

$$\alpha = -\frac{1}{T}\frac{\frac{L}{L_0} - \left(\frac{L_0}{L}\right)^2}{\frac{L}{L_0} + \frac{2L_0^2}{L^2}}.$$

[7 marks]

b) For the elastic band of part a), the Gibbs free energy is defined as $G = U - TS - \mathcal{F}L$.

i) The incremental work done on an elastic band extended by an infinitesimal length *dL* is given by $dW = \mathcal{F}dL$. Write down the thermodynamic identity for the elastic band.

ii) Using the above definition of *G* and the thermodynamic identity for an elastic band find the natural variables of *G***.**

iii) Find the relationships between the partial differentials of *G* with respect to each of the two natural variables and two other thermodynamic variables.

iv) From **ii)** and **iii)** find a Maxwell relation between partial differentials of *S* and *L* with respect to the natural variables of *G***.** **[8 marks]**

c) The effect of surface tension means that work has to be done in creating new area in, for example, a soap film. The infinitesimal work done in creating that new area is given by; $dW = \Gamma dA$; where Γ is the surface tension of the film.

i) Write down an expression for the infinitesimal, isothermal, work done on a soap bubble containing gas at pressure *P*.

ii) The soap bubble, at temperature, *T*, is expanded isothermally from an initial radius, $\mathcal{R}_i$, to a final radius, $\mathcal{R}_f$,. Write down an expression for the work done in this process. You may assume that the gas that inflates the bubble may be treated as an ideal gas.

iii) Write down an expression for the infinitesimal change in the Helmholtz free energy, *F*, of this soap bubble.

iv) Consider the soap bubble at constant temperature and containing a constant quantity of gas. By using the result of part iii) and considering the equilibrium condition on *F* show that the pressure of the gas inside the bubble is given by $P = \frac{2}{\mathcal{R}}\Gamma$. **[10 marks]**

Question B2

a)

i) Write down an expression for the incremental entropy change, dS, of a system as it goes from an initial equilibrium state to a final equilibrium state in terms of reversible heat transfer, dQ_R, and the temperature, T, at which it occurs. Explain the sign convention.

ii) 5 kg of ice from a freezer at -5^{o}C is placed in a bucket in the garden where the temperature is 20^{o}C. Calculate the entropy change of the ice/water, ***including sign***, after all the ice has melted and the water has come to thermal equilibrium.

iii) Calculate the entropy change of the garden (due to the processes occurring with the ice/water)

iv) Demonstrate with your answers to ii) and iii) that the second law of thermodynamics is obeyed.

[10 marks]

b)

i) By manipulating the thermodynamic identity for a P-V-T system and the equation of state show that for one mole of an ideal rigid diatomic gas the entropy change, as volume and temperature are changed, is given by;

$$\Delta S = S(V_f, T_f) - S(V_i, T_i) = \frac{5}{2}R\ln\left[\frac{T_f}{T_i}\right] + R\ln\left[\frac{V_f}{V_i}\right].$$

ii) An ideal rigid diatomic gas is expanded isothermally from V_i to $3V_i$ Calculate a figure for the entropy change of the gas ***including the sign***.

iii) An ideal rigid diatomic gas is cooled in an isochoric process from temperature T_i to $0.5T_i$. Calculate a figure for the entropy change of the gas ***including the sign***.

iv) Demonstrate that for an ideal gas undergoing an adiabatic expansion from initial volume V_i to final volume V_f there is zero change in entropy.

[10 marks]

c) The equation of state for one mole of a monatomic Van der Waals gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

and the internal energy is given by

$$U_{VdW} = \frac{3}{2}RT - \frac{a}{V}.$$

Show that the change in entropy for a Van der Waals gas is given by;

$$\Delta S = \frac{3}{2}R\ln\left[\frac{T_f}{T_i}\right] + R\ln\left[\frac{V_f - b}{V_i - b}\right].$$

[5 marks]

Question B3

a)

i) In an isothermal process one mole of a ***non-rigid diatomic*** ideal gas at temperature, T, is compressed from an initial volume, V_i, to a final volume, $V_i/3$. What is the work done in this process in terms of T and V_i ? ***Be careful to include the sign*** and state whether the work was done by or on the gas.

ii) In an isobaric process one mole of a ***rigid diatomic*** ideal gas at pressure, P_i, is expanded from initial volume, V_i, to a final volume, $3V_i$. Write down the work done during this process in terms of P_i and V_i. ***Be careful to include the sign***.

iii) In an isochoric process one mole of a ***monatomic*** ideal gas of volume, V_i, has its pressure increased from initial pressure, P_i, to a final pressure, $3P_i$. What is the heat transfer during this process in terms of P_i and V_i? ***Be careful to include the sign*** and state whether the heat was absorbed or released by the gas.

[6 marks]

b) A reversible engine is constructed from a ***rigid diatomic*** gas that operates in the following cycle; it undergoes an isobaric expansion from $\mathbf{1} \rightarrow \mathbf{2}$ starting at pressure P_1 and volume V_1 going to volume V_2. It then undergoes an isochoric process from $\mathbf{2} \rightarrow \mathbf{3}$ starting at V_2 and going from P_1 to P_3 before returning to its initial state in an adiabatic process $\mathbf{3} \rightarrow \mathbf{1}$.

i) Construct a **fully labelled** P-V diagram of this engine cycle including any heat flows and their direction.

ii) Calculate the heat flows in the isobaric process and in the isochoric process in terms of P and V.

iii) Define the engine efficiency, η_E, for a heat engine.

iv) Show that for this cycle the efficiency is given by;

$$\eta_E = 1 - \frac{5}{7}\frac{1 - P_3/P_1}{1 - \left(P_3/P_1\right)^{5/7}}.$$

[9 marks]

c) A heat engine operates between a tank containing 1000 m^3 of water and a river at a constant temperature of 10°C. The temperature of the tank is initially 100°C. The density of water is 1 gm/cc.

i) What is the maximum amount of work which the heat engine can perform?

ii) What is the overall conversion efficiency?

[10 marks]

HINT: The hot reservoir in this problem is of finite size only, therefore, it will be cooled by the heat extracted from it.

Question B4

a)

i) 3×10^{-3} moles of molecular oxygen are confined to a volume of 300 cm^3 and are found to be at a pressure of 5 atmospheres. The molecular mass of O_2 is 32 amu. Treating the O_2 as an ideal gas, calculate the root mean square velocity, v_{rms}, of the molecules.

ii) The temperature of the sun's photosphere is 6000^oC and it consists mainly of Hydrogen with mass 1 amu and Helium with mass 4 amu. Calculate the root mean square velocity, v_{rms}, of the Hydrogen and Helium atoms.

[5 marks]

b) For a gas of molecules of mass *m* at temperature *T* the Maxwell Boltzmann velocity distribution function is given by;

$$f(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left[-\frac{mv^2}{2k_B T}\right].$$

Where *v* is the magnitude of the velocity in a given direction.

i) What is the Maxwell speed distribution?

ii) Demonstrate that the most probable or mode speed, v_m, is given by;

$$v_m = \sqrt{\frac{2k_B T}{m}}.$$

iii) The mean speed is given by;

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}}.$$

If an ideal gas is at a pressure *P* and a temperature *T*, what is the mean flux, Φ, of gas atoms crossing unit area in unit time in terms of these thermodynamic parameters?

[10 marks]

c)

i) A vacuum chamber is designed to keep silicon wafers clean and the pressure of gas is kept as low as possible in order to maintain a clean surface. The coverage of a surface requires about 10^{19} molecules per m^2. Nitrogen gas is the majority species in the atmosphere so considering N_2 alone, estimate what maximum pressure may be tolerated in order to deposit less than one monolayer of nitrogen atoms per hour? The mass of N_2 is 14 amu. You may assume that if a molecule hits the surface it sticks.

ii) The planet Venus has a mass M = 4.87×10^{24} kg and a radius of 6000km. At what temperature would hydrogen need to be on the surface of Venus such that its mean velocity was just equal to the escape velocity?

[10 marks]

End of Paper - An appendix of 1 page follows

DATA SHEET

You may wish to use some of the following data.

k_B	= Boltzmann's constant	= 1.38×10^{-23} J K^{-1}
N_A	= Avagadro's number	= 6.02×10^{23} mol^{-1}
R	= Gas constant	= 8.31 J mol^{-1} K^{-1}
P_{atm}	= Atmospheric pressure	= 1atm = 1.01×10^{5} Pa
T_S	= Ice point of water	= 273.15 K
c_P	= Specific heat of water at constant pressure	= 4.2×10^{3} J K^{-1} kg^{-1}
c_P^{Ice}	= Specific heat of ice at constant pressure	= 2.1×10^{3} J kg^{-1}
l^{SL}	= Latent heat of melting ice	= 3.33×10^{5} J kg^{-1}
1 *amu*	= One atomic mass unit	= 1.66×10^{-27} kg