Hidden dimensions of Spacetime in String Theory

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1. Dimension of objects and space

A long thin thread. To specify the location of a point along the thread, it suffices to give one number -- the distance from one end. We can call this number the *x-coordinate* of the point.

Observe the surface of a table.

To specify the location of a point : two numbers

Displacements along two axes

(x , y) coordinates of the point.

Now observe the space around you -- in the whole room.

Specify the location of a point : Three numbers

Displacements along three axes from a chosen origin.

(x, y, z) coordinates of the point.

The *number of coordinates* required to specify a point in a space defines the *dimension* of the space.

So:

A thin thread is a *One-dimensional* space.

The surface of a table is a *Two-Dimensional* space.

And the space in a room, on earth , in the solar system, or the universe we have observed is a *Three-Dimensional* space.

The Mathematical description of higher dimensional space is simply in terms of a larger set of coordinates.

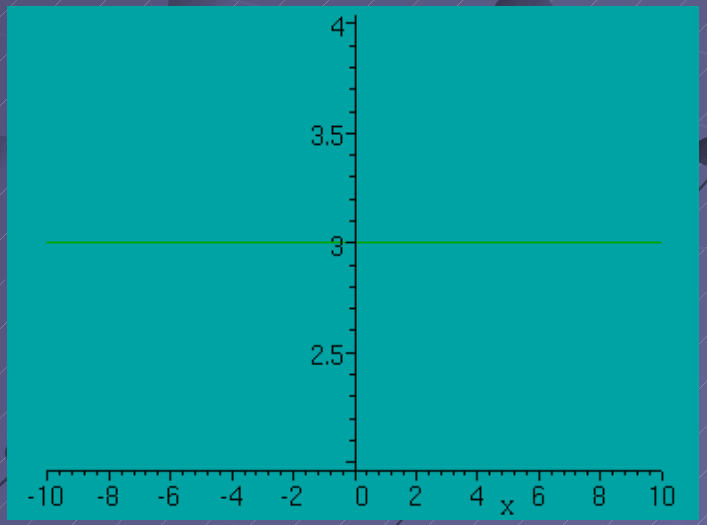
For example (x , y, z , w) will describe *four space dimensions.*

In *String Theory* it becomes necessary to think about 9 (!!) space dimensions (x_1 , x_2, ... , x_9)

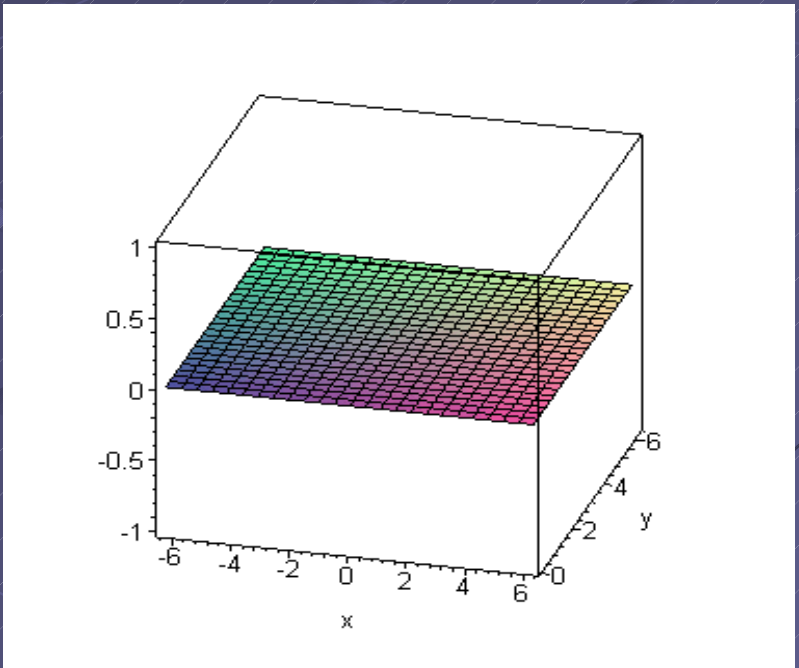
1.4 Space embedded in higher dimensional space

Cartesian coordinates and equations we can write in terms of them are useful in describing spaces of some dimension embedded in a space of higher dimension.

For example consider the two-dimensional space with coordinates (x , y). The equation y = 3 describes a line. Points on the line have coordinates (x , 3). Moving along this line changes the value of x. Thus the equation specifies a one-dimensional line "living in" or embedded in two dimensional space.



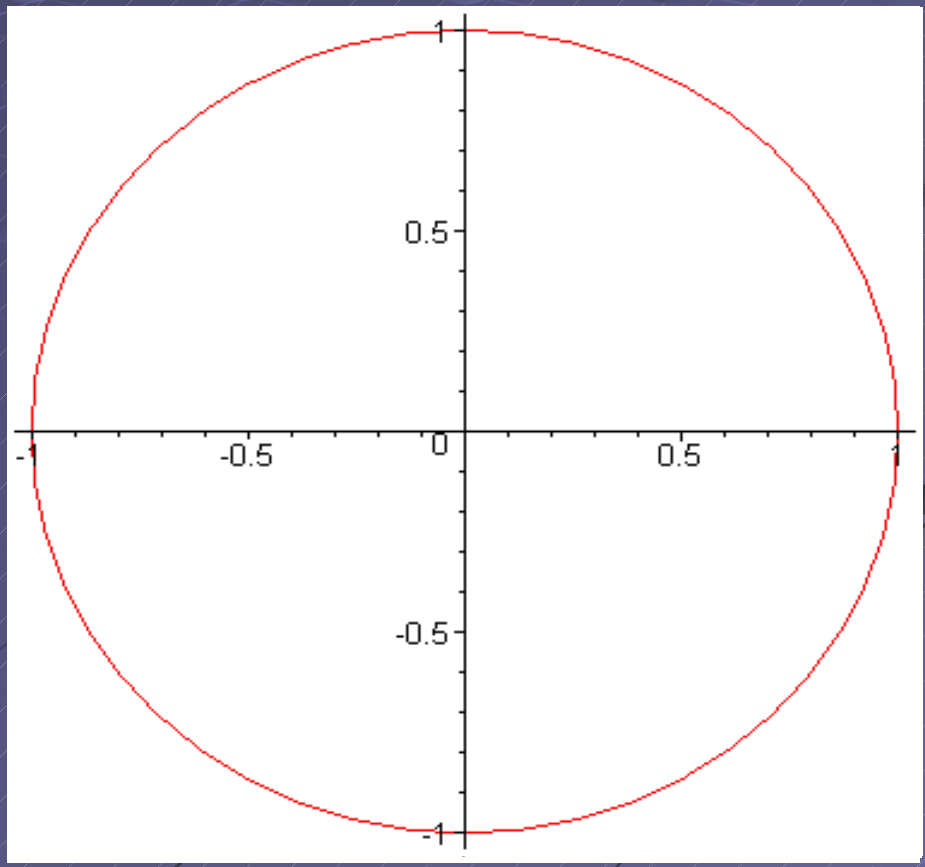
Similarly if we start with a three-dimensional space the equation $z = 0$ restricts us to a two-dimensional plane $(x , y , 0)$.
Physically this could be the description of a surface such as the ceiling, the black board, the plane of the Earth's orbit …. in space.
This allows a simple generalization. Our world described by (x , y, z) could be a three-dimensional spatial sheet in four spatial dimensions (x, y, z, w) with the restriction $w = 0$. This is the idea of the Brane-world which we will come back to.



Many interesting geometries can be described mathematically by specifying an equation.

A circle : $x^2 + y^2 = 1.$

The circle is the set of points at a fixed distance from the origin in two dimensions.

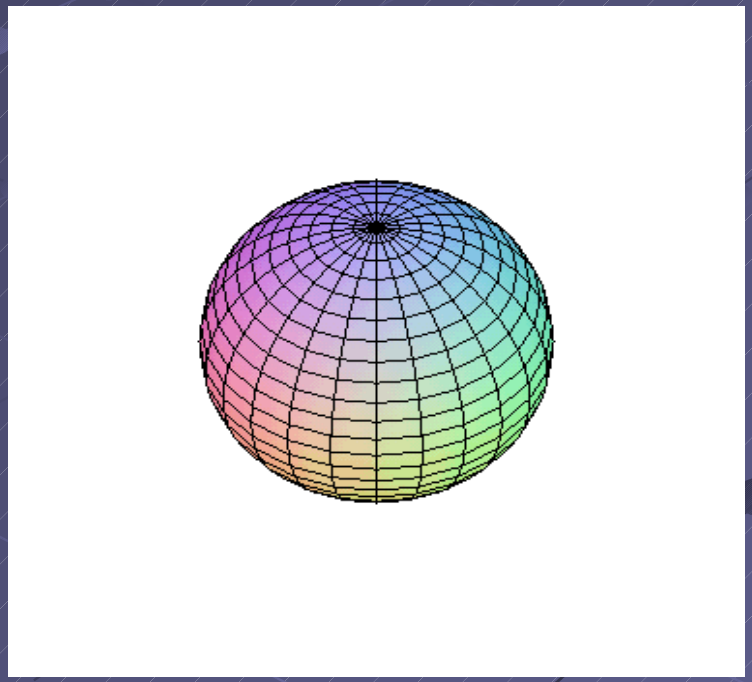


This can be understood using the Pythagoras theorem , which you have seen in geometry.

A sphere : $x^2 + y^2 + z^2 = 1$

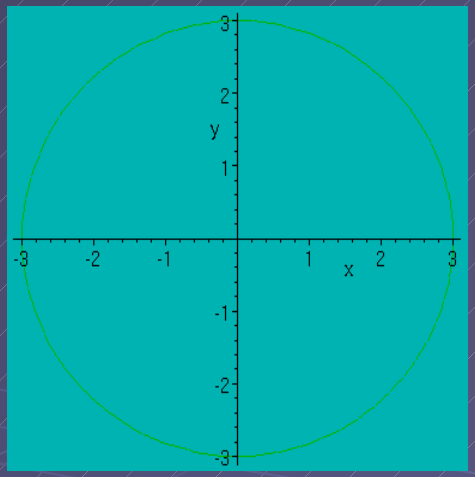
Equation of a FOOTBALL

This is a mathematical expression of the fact that the sphere is the set of points in three dimensional space which is at a fixed distance from the origin. Again Pythagoras theorem is useful.

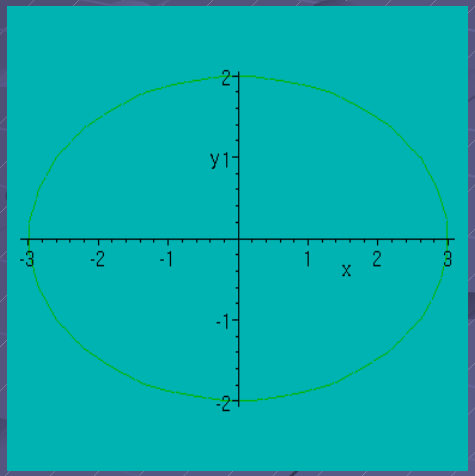


What if we play with the equations…?

Circle: $x^2/9 + y^2/9 = 1$

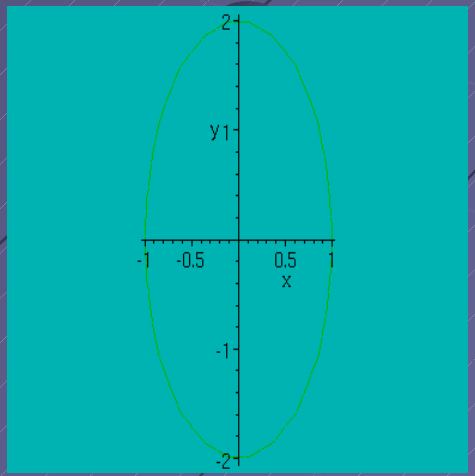


$x^2/9 + y^2/4 = 1$



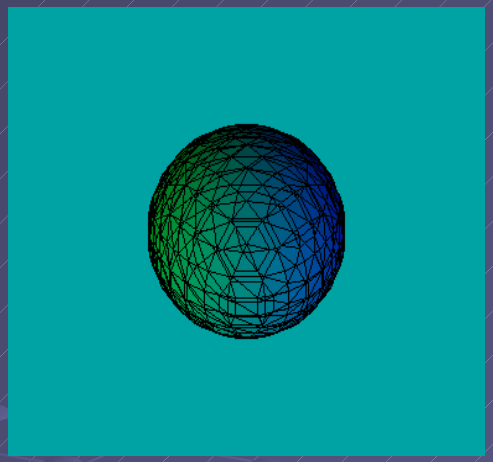
Ellipses:

$x^2 + y^2/4 = 1$

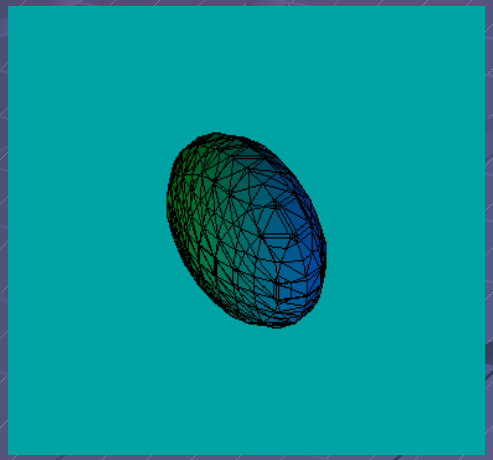


What about the sphere..?

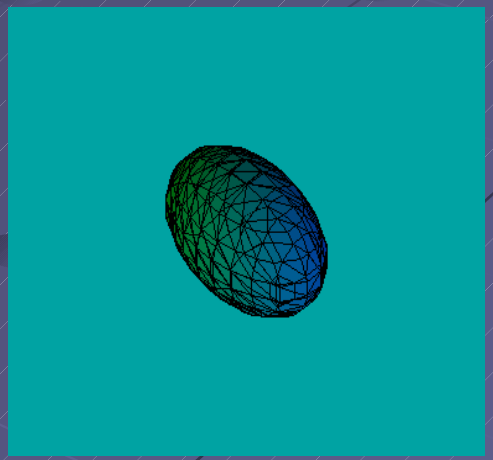
Sphere : $x^2/25 + y^2/25 + z^2/25 = 1$



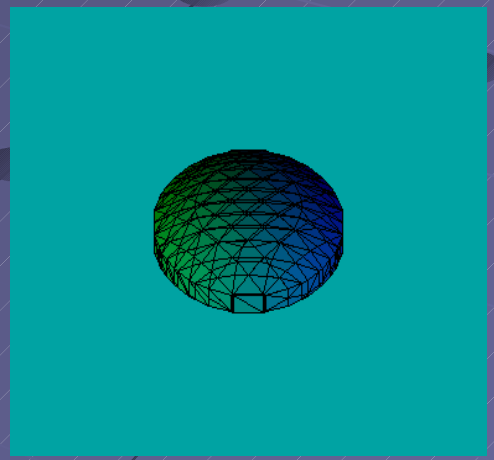
$x^2/8 + y^2/25 + z^2/25 = 1$



Ellipsoids: $x^2/25 + y^2/25 + z^2/4 = 1$

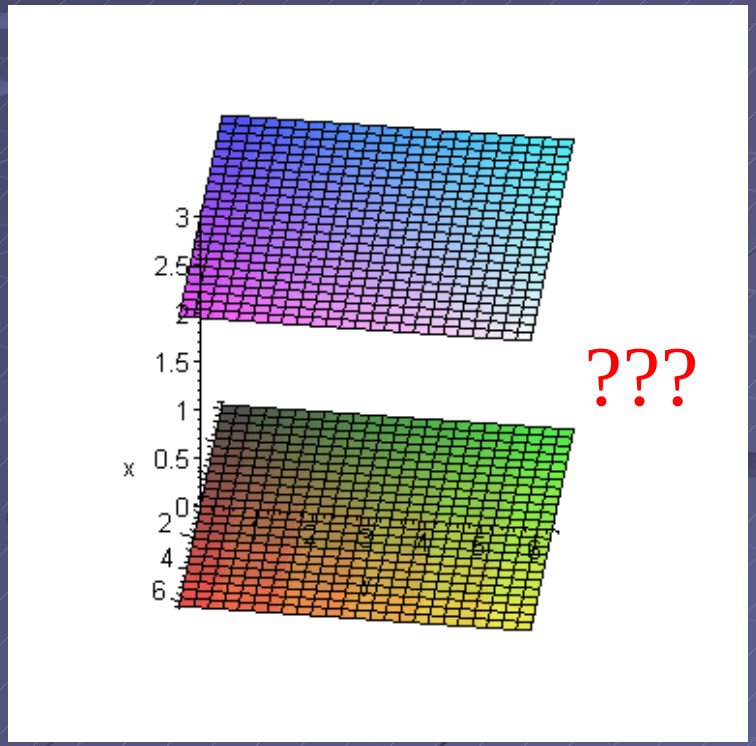


$x^2/9 + y^2/25 + z^2/16 = 1$



2. BRANE WORLDS, Particles and Forces.

The Brane World idea is that our the three-space dimensions of the world form a sheet or Brane living in a world with more dimensions. In the simplest such model we have one extra dimension stretching out away from the Brane.



If there is an extra dimension stretching out away from our world, why can't we see it ? We certainly see objects approaching us or departing from us in any of the three familiar directions. So why do we not have objects travelling away from us in the extra direction.

The answer comes from String Theory.

String Theory is the idea that tiny strings form the underlying constituents of all matter and force.

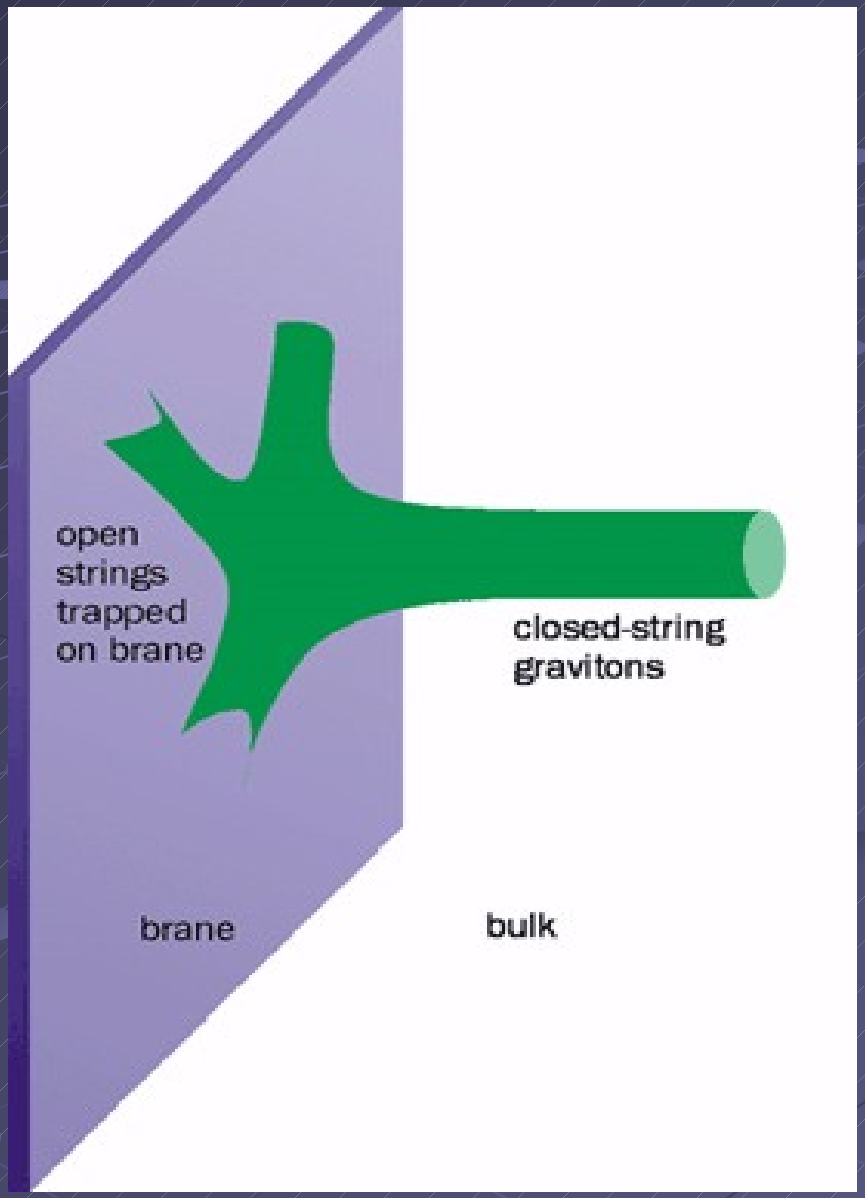
How Tiny ? Tinier than atoms, protons, quarks, they could be as small as 10^{-33} cm. Different vibrations of these strings give rise to electrons, protons, atoms as well as the photons of light which are messengers of the electric force and the gravitons which mediate the gravitational force.

Strings naturally propagate in nine space dimensions. Lots can happen in these nine dimensions. In these nine dimensions there can live branes having 1, 2, 3, 4 or more dimensions. String Theory tells us a lot about the bevaviour of these branes.

String theory shows that such brane worlds can exist and that we do not see objects which are separated from us in this extra direction because light is confined to live on our brane. The same mechanism that confines light can confine other microscopic forces such as the strong force and the weak force.

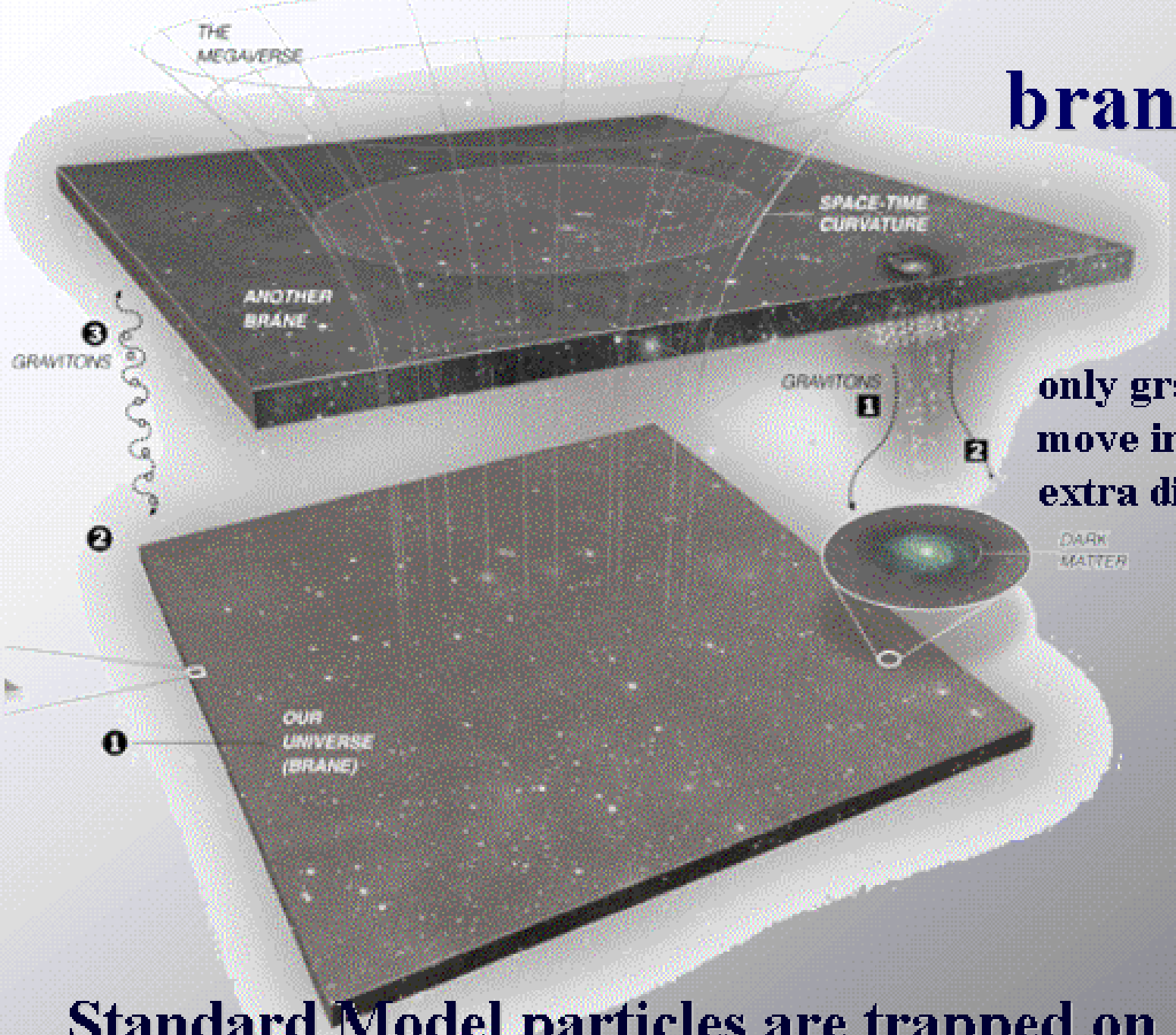
But if all forces are confined to our brane and nothing could ever be observed from the extra dimensions, it is just as if they were never there, isn't it ?…

…Not exactly. Because String theory shows that while the particles of light and strong and weak forces come from open strings, the particles of gravity come from closed strings. The open strings are confined to branes whereas the closed strings can escape.



This shows that while microscopic forces can be confined on our world-brane gravity can allow us to experience extra dimensions.

braneworlds



only gravitons and exotics move in the "bulk" of the extra dimensional universe

Standard Model particles are trapped on a brane and can't move in the extra dimensions

BREAK !!!

3.1 Time as an extra dimension

If we specify the time where an event happens, as well as the spatial location, we have *four numbers* describing the event

$$(x , y , z , t)$$

We may say that the observable world has *four dimensions of space-time*, three of space and one of time.

Central in special relativity – which replaces ordinary Newtonian mechanics for objects travelling close to the speed of light. For example, just the way we collect space and time in an array of 4 numbers, we are lead to collect energy and momentum into an array of four numbers

$$(p_x, p_y, p_z, E/c)$$

And…

$$M^2 c^2 = (E/c)^2 - p_x^2 - p_y^2 - p_z^2$$

In higher dimensions for a massless particle

$$0 = (E/c)^2 - p_x^2 - p_y^2 - p_z^2 - p_w^2$$

The extra momentum looks like a mass in three dimensions

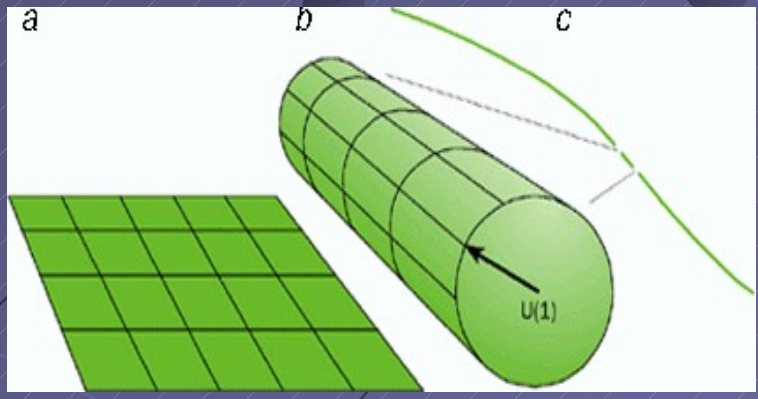
3.2 Tiny Hidden dimensions

In the Brane world picture described above the extra dimensions are large. String theory also allows the possibility of tiny extra dimensions without any branes, which are curled up in shapes like circles, spheres or such.

If tiny extra dimensions are real, then why have we not seen them ?

Because our eyes and all other instruments we use to observe the world do not have the sensitivity to observe such tiny structures.

If for example the tiny dimensions were 10^{-33} *cm* in length, we would never have seen them.



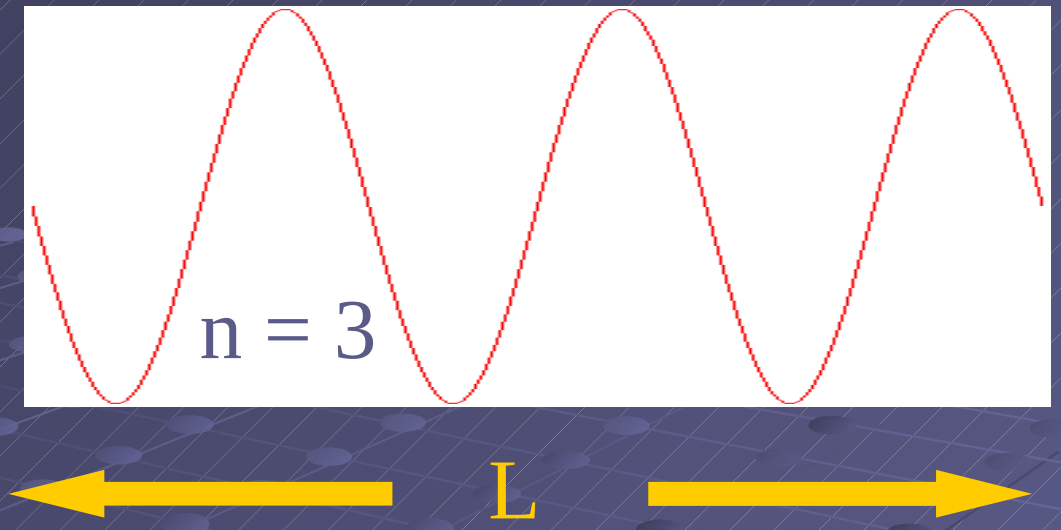
So where do we live in these extra dimensions ?

The startling answer, which is only possible because the world is described at a microscopic level by Quantum Mechanics and not Newtonian mechanics :

We live everywhere !!

In Quantum mechanics particles are described, not by a position and velocity but by a *wavefunction* which gives the *probabilities* of different locations. Suppose there is one extra dimension in the shape of a circle. Then the wave describing the probability of finding any particle or collection of particles (like us !) has to fit on the circle. The wave has To have a wavelength which is an integer fraction of the circumference of the circle.

$L = \lambda\ n$



The more wavy it is, the higher the momentum. This follows from the De Broglie Equation : $p = h / \lambda$, where λ is the wavelength and h is Planck's constant.

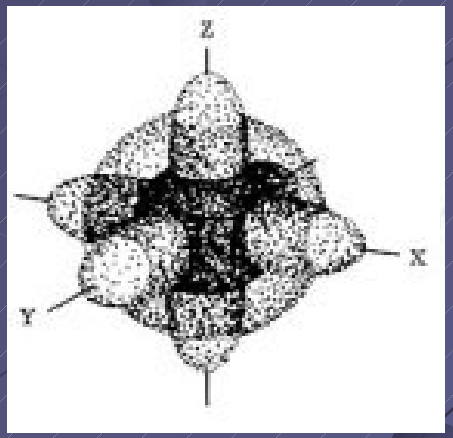
More wavy means smaller λ hence higher momentum!

A particle carrying such momentum along the circle has a lot of energy even if it is at rest in the visible directions of space. This rest energy contributes to the mass of the particle. Hence, waveforms which wave a lot in the hidden dimensions correspond to very heavy particles which might be seen in high energy accelerators.

Ordinary particles such as electrons have zero momentum along the hidden direction and they have waveforms which do not wave a lot in the extra dimension ...

…in fact they are just constant.

Constant wavefunctions indicate that the probability of finding these particles is the same at any location in the extra dimension. It is in this sense that electrons, protons and things built from them, like ourselves, live at all locations in the small hidden dimension.



Summary so far :

String theory is a framework for describing microscopic particles
And forces as well as gravitons and the force of gravity.

String theory is compatible with Special relativity of Einstein
And quantum physics of Bohr , Heisenberg etc.

Its simplest forms predict that space-time is 10 dimensional.

It raises many questions.

What is the typical size of the strings ?

They can't be too big. We would have seen them already !!

A first guess is the Planck length 10^{-33} cm

What is the size of the extra dimensions ?

A first guess : Also Planck length.

New ideas suggest other consistent possibilities.

But they could be bigger. If sufficiently big, their
Effects could be seen in high energy particle collisions
At the LHC in CERN.

LHC is 27 Km in circumference.

Will collide protons against protons at highest energies ever reached in accelerators.

More on particle physics, collisions, LHC at the CERN Web-site :

http://public.web.cern.ch/public/

More on string theory research at Queen Mary, University of London

http://www.strings.ph.qmw.ac.uk/

Getting started :Studying physics at University

http://www.ph.qmul.ac.uk/