Conserved Quantities in Lemaître-Tolman-Bondi Cosmology

Alex Leithes - Blackboard Talk Outline

Conserved Quantities in Perturbed LTB

- $\zeta_{\rm SMTP}$ Evolution Equation:

$$\dot{\zeta}_{\rm SMTP} = \frac{H_X + 2H_Y}{3\bar{\rho}} \delta P_{\rm nad}$$

- Valid on all scales.
- For barotropic fluids $\dot{\zeta}_{\rm SMTP} = 0$

From arXiv:1403.7661 (submitted to CQG) by AL and Karim A. Malik

Queen Mary University of London

Overview

Conserved Quantities in Perturbed LTB

- $\zeta_{\rm SMTP}$ Evolution Equation:

$$\dot{\zeta}_{\rm SMTP} = \frac{H_X + 2H_Y}{3\bar{\rho}} \delta P_{\rm nad}$$

- "I will leave this equation on the board from the beginning of this talk, as it is the main result and I'd hate to run out of time without getting to it."

Contents

- Why Perturb LTB Cosmology?
- The Standard Model of Cosmology - Flat FRW
- Conserved Quantities in Perturbed LTB

Why Perturb LTB Cosmology?

Why Perturb LTB Cosmology?

- Recent observations (e.g. SN1a) suggest late time accelerated expansion - driven by Dark Energy
- Inhomogeneous Cosmologies explain observations through inhomogeneous expansion not acceleration - no Dark Energy
- Many possible inhomogeneous cosmologies; LTB type models still actively researched (simplest model, toy model)
- Ongoing work interpreting observations (galaxy surveys, large scale structure surveys, CMB) and other redshift dependent observations in context of LTB background

The Standard Model of Cosmology - Flat FRW

"Starting with the standard."
FRW: Maximally symmetric spatial section - expansion time dependent only

The Standard Model of Cosmology - Flat FRW

- Background metric:

$$ds^2 = -dt^2 + a^2\delta_{ij}dx^i dx^j$$

- Perturbed metric:

$$ds^2 = -(1+2\Phi)dt^2 + 2aB_i dx^i dt + a^2(\delta_{ij} + 2C_{ij})dx^i dx^j$$

with scalar, vector and tensor perturbations[a]

[a] e.g. Bardeen 1980

The Standard Model of Cosmology - Flat FRW

The Standard Model of Cosmology - Flat FRW

- Further decomposition of 3-spatial perturbations gives curvature perturbation ψ, identified with the intrinsic scalar curvature:

$$C_{ij} = E_{,ij} - \psi\delta_{ij} + vector + tensor \quad quantities^*$$

* On 3-spatial hypersurfaces

The Standard Model of Cosmology - Flat FRW

Constructing Gauge Invariant Quantities

- Splitting quantities into background + perturbation: no longer covariant - gauge dependent; construct gauge invariant quantities
- General gauge transformations:

$$\widetilde{\delta \mathbf{T}} = \delta \mathbf{T} + \pounds_{\delta x^\mu} \bar{\mathbf{T}}$$

- Tilde denotes new coordinates

$$\widetilde{x^\mu} = x^\mu + \delta x^\mu$$

  bar denotes background.

- Key quantities gauge transformations:

$$\begin{aligned} \widetilde{\psi_{\rm FRW}} &= \psi_{\rm FRW} + \frac{\dot{a}}{a}\delta t \\ \widetilde{\delta\rho_{\rm FRW}} &= \delta\rho_{\rm FRW} + \dot{\bar{\rho}}\delta t \end{aligned}$$

The Standard Model of Cosmology - Flat FRW

Constructing Gauge Invariant Quantities

- Gauge choice: uniform density hypersurfaces, $\widetilde{\delta\rho_{\mathrm{FRW}}} = 0$

$$\delta t = -\frac{\delta\rho_{\mathrm{FRW}}}{\dot{\bar{\rho}}}$$

Get gauge invariant curvature perturbation

$$-\zeta \equiv \psi_{\mathrm{FRW}} + \frac{\dot{a}/a}{\dot{\bar{\rho}}}\delta\rho$$

- Evolution equations for ζ from time derivative, $\delta\rho$ from energy conservation $\nabla_\mu T^{\mu\nu} = 0$...

  ζ conserved in large scale limit - conserved perturbed quantities allow easily relate early to late times (e.g. curvature/physics early time relates to density/observables late time)

Conserved Quantities in Perturbed LTB

"Compare with our inhomogeneous model."
LTB: Spherically symmetric spatial section - expansion time and r coordinate dependant (not θ, ϕ)[1]

Perturbed LTB

- Background metric:

$$ds^2 = -dt^2 + X^2(r,t)dr^2 + Y^2(r,t)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Perturbed Metric:

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i d\mathcal{X}^i dt + (\delta_{ij} + 2C_{ij})d\mathcal{X}^i d\mathcal{X}^j$$

Where $d\mathcal{X}^i = [Xdr, Yd\theta, Y\sin\theta d\phi]$ and we reserve dx^i for $[dr, d\theta, d\phi]$

[1] Bondi 1947

Conserved Quantities in Perturbed LTB

Perturbed LTB

- We have performed 1+3 decomposition into time and spatial sections of metric
- Decomposition of perturbations not completely straightforward without use of methods like spherical harmonic decomposition but...[a]
- Our undecomposed perturbations give simpler expressions, easing constructing conserved quantities.

[a] e.g. Clarkson, Clifton, February 2009

Conserved Quantities in Perturbed LTB

Perturbed LTB

- Background Energy Conservation:

$$\dot{\rho} + \rho(H_X + 2H_Y) = 0\,, \qquad H_X = \frac{\dot{X}}{X}\,, \qquad H_Y = \frac{\dot{Y}}{Y}$$

- Perturbed Energy Conservation:

$$\begin{aligned} \delta\dot{\rho} \quad &+ \quad (\delta\rho + \delta P)\,(H_X + 2H_Y) + \bar{\rho}' v^r \\ &+ \quad \bar{\rho}\Big(\dot{C}_{rr} + \dot{C}_{\theta\theta} + \dot{C}_{\phi\phi} + \partial_r v^r + \partial_\theta v^\theta + \partial_\phi v^\phi \\ &+ \quad \left[\frac{X'}{X} + 2\frac{Y'}{Y}\right] v^r + \cot\theta v^\theta\Big) = 0 \end{aligned}$$

- Convenient to define spatial metric perturbation:

$$3\psi = \frac{1}{2}\delta g^k_k = C_{rr} + C_{\theta\theta} + C_{\phi\phi}$$

Conserved Quantities in Perturbed LTB

Constructing Gauge Invariant Quantities

- ψ transformation behaviour:

$$3\tilde{\psi} = 3\psi + \left[\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y}\right]\delta t + \left[\frac{X'}{X} + 2\frac{Y'}{Y}\right]\delta r + \partial_i \delta x^i + \delta\theta \cot\theta \,,$$

- Gauge choices; uniform density:

$$\delta t\Big|_{\delta\tilde{\rho}=0} = -\frac{1}{\dot{\bar{\rho}}}\left[\delta\rho + \bar{\rho}'\delta r\right] .$$

comoving:

$$\delta x^i = \int v^i dt \,.$$

Conserved Quantities in Perturbed LTB

Constructing Gauge Invariant Quantities

- Gives gauge invariant **Spatial Metric Trace Perturbation (SMTP)** on comoving, uniform density hypersurfaces:

$$
\begin{aligned}
-\zeta_{\mathbf{SMTP}} &= \psi + \frac{\delta\rho}{3\bar{\rho}} + \frac{1}{3}\Bigg\{ \left(\frac{X'}{X} + 2\frac{Y'}{Y} + \frac{\bar{\rho}'}{\bar{\rho}} \right) \int v^r dt \\
&+ \partial_r \int v^r dt + \partial_\theta \int v^\theta dt + \partial_\phi \int v^\phi dt + \cot\theta \int v^\theta dt \Bigg\}
\end{aligned}
$$

Conserved Quantities in Perturbed LTB

Constructing Gauge Invariant Quantities

- Get gauge invariant density perturbation on uniform curvature hypersurfaces

$$
\begin{aligned}
\delta\tilde{\rho}\Big|_{\psi=0} &= \delta\rho + \bar{\rho}\left\{3\psi + \left(\frac{X'}{X} + 2\frac{Y'}{Y} + \frac{\bar{\rho}'}{\bar{\rho}}\right)\int v^r\, dt \right. \\
&+ \left. \partial_r \int v^r\, dt + \partial_\theta \int v^\theta\, dt + \partial_\phi \int v^\phi\, dt + \cot\theta \int v^\theta\, dt \right\}
\end{aligned}
$$

- May be related to ζ_{SMTP} as

$$
\delta\tilde{\rho}\Big|_{\psi=0} = -3\bar{\rho}\zeta_{\mathrm{SMTP}}
$$

Conserved Quantities in Perturbed LTB

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Conclusion and Further Research

- $$\dot{\zeta}_{\mathbf{SMTP}} = \frac{H_X + 2H_Y}{3\bar{\rho}} \delta P_{\mathbf{nad}}$$
- Research already extended to other spacetimes.
  i.e.$\dot{\zeta}_{\mathbf{SMTP}}$ already extended to Lemaitre and FRW
- Potential wider use of $\zeta_{\mathbf{SMTP}}$ in inhomogeneous spacetimes generally versus standard FRW model.

arXiv:1403.7661