

Observing the normal Zeeman effect in transverse and longitudinal configuration Spectroscopy with a Fabry-Perot etalon

Principle

The objective of this experiment is to observe the normal Zeeman effect in the light from a cadmium lamp and to perform quantitative measurements to determine value of the Bohr magnetron.

There are two parts to this experiment.

1. To perform qualitative observations of the Zeeman effect including:
 - Observing the line triplet for the normal transverse Zeeman effect.
 - Determining the polarization state of the triplet components.
 - Observing the line doublet for the normal longitudinal Zeeman effect.
 - Determining the polarization state of the doublet components.
2. To perform quantitative measurements on the normal transverse Zeeman effect and to calculate the value of the Bohr magnetron

Theory

Normal Zeeman effect

The Zeeman effect is the name for the splitting of atomic energy levels or spectral lines due to the action of an external magnetic field. The effect was first predicted by H. A. Lorentz in 1895 as part of his classic theory of the electron, and experimentally confirmed some years later by P. Zeeman. Zeeman observed a line triplet instead of a single spectral line at right angles to a magnetic field, and a line doublet parallel to the magnetic field. Later, more complex splitting of spectral lines were observed, which became known as the anomalous Zeeman effect. To explain this phenomenon, Goudsmit and Uhlenbeck first introduced the hypothesis of electron spin in 1925. Ultimately, it became apparent that the anomalous Zeeman effect was actually the rule and the "normal" Zeeman effect the exception.

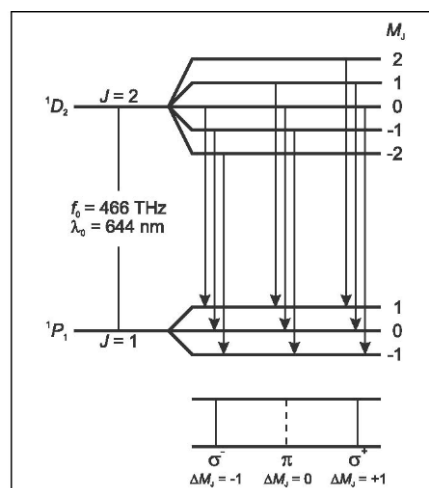


Figure 1. Level splitting and transitions of the normal Zeeman effect in Cadmium

The normal Zeeman effect only occurs at the transitions between atomic states with the total spin $S = 0$. The total angular momentum $J = L + S$ of a state is then pure orbital angular momentum ($J = L$). For the corresponding magnetic moment, μ , we can simply say that,

$$\mu = \frac{\mu_B}{\hbar} J \quad (1)$$

where

$$\mu_B = \frac{\hbar e}{-2m_e} \quad (2)$$

(μ_B = Bohr magneton, m_e = mass of electron, e = elementary charge, $\hbar = h/2\pi$, h = Planck's constant).

In an external magnetic field, B , the magnetic moment has the energy

$$E = -\mu B \quad (3)$$

The angular-momentum component in the direction of the magnetic field can have the values

$$J_z = M_J \hbar \quad \text{with} \quad M_J = J, J-1, \dots, -J \quad (4)$$

Therefore, the term with the angular momentum, J is split into $2J + 1$ equidistant Zeeman components which differ by the value of M_J . The energy interval of the adjacent components M_J , M_{J+1} is

$$\Delta E = \mu_B B \quad (5)$$

We can observe the normal Zeeman effect e.g. in the red spectral line of cadmium ($\lambda_0 = 643.8$ nm, $f_0 = 465.7$ THz). It corresponds to the transition 1D_2 ($J = 2, S = 0$) \rightarrow 1P_1 ($J = 1, S = 0$) of an electron of the fifth shell (see fig. 1). In the magnetic field, the 1D_2 level splits into five Zeeman components, and the level 1P_1 splits into three Zeeman components having the spacing calculated using equation 5.

Optical transitions between these levels are only possible in the form of electrical dipole radiation. The following selection rules apply for the magnetic quantum numbers M_J of the states involved:

$$\Delta M_J \begin{cases} = \pm 1 & \text{for } \sigma \text{ components} \\ = 0 & \text{for } \pi \text{ components} \end{cases} \quad (6)$$

Thus, we observe a total of three spectral lines (see fig. 1); the π component is not shifted and the two σ components are shifted by

$$\Delta f = \pm \frac{\Delta E}{h} \quad (7)$$

with respect to the original frequency. In this equation, ΔE is the equidistant energy split calculated in equation 5.

Angular distribution and polarization

Depending on the angular momentum component, ΔM_J , in the direction of the magnetic field, the emitted photons exhibit different angular distributions. Figure 2 shows the angular distributions in the form of two-dimensional polar diagrams. They can be observed experimentally, as the magnetic field is characterized by a common axis for all cadmium atoms.

In classical terms, the case $\Delta M_J = 0$ corresponds to an infinitesimal dipole oscillating parallel to the magnetic field. No photons are emitted in the direction of the magnetic field, i.e. the π -component cannot be observed parallel to the magnetic field. The light emitted perpendicular to the magnetic field is linearly polarized, whereby the E -vector oscillates in the direction of the dipole and parallel to the magnetic field (see fig. 3)

Conversely, in the case $\Delta M_J = \pm 1$ photons can be emitted both parallel and normal to the direction of the magnetic field. In classical terms, this case corresponds to two dipoles oscillating parallel to the magnetic field with a phase difference of 90° . The superposition of the two dipoles produces a circulating current. Thus, in the direction of the magnetic field, circularly polarized light is emitted; in the positive direction, it is clockwise-circular for $\Delta M_J = +1$ and anticlockwise-circular for $\Delta M_J = -1$ (see fig. 3).

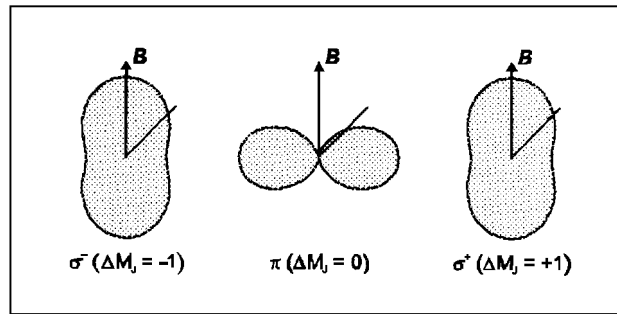


Figure 2. Angular distributions of the electrical dipole radiation (ΔM_J : angular-momentum components of the emitted photons in the direction of the magnetic field)

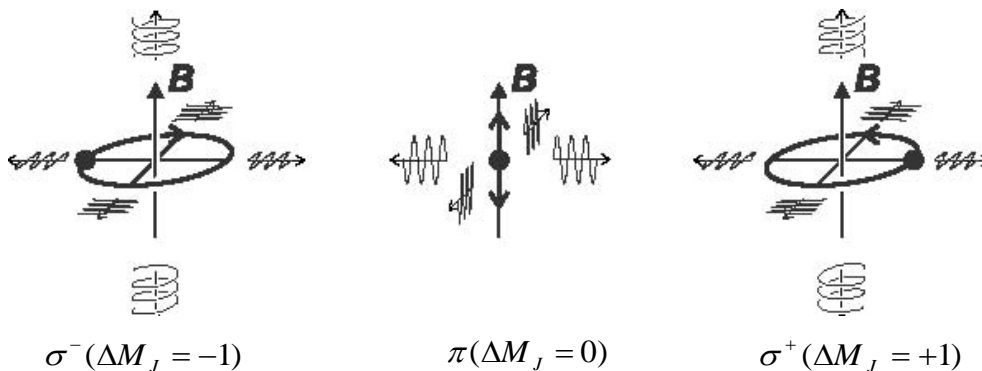


Figure 3. Schematic representation of the polarization of the Zeeman components (ΔM_J : angular-momentum components of the emitted photons in the direction of the magnetic field)

Spectroscopy of the Zeeman components

The Zeeman effect enables spectroscopic separation of the differently polarized components. However, to demonstrate the shift we require an apparatus with extremely high spectral resolution, as the frequency of the two σ components of the red cadmium line are shifted, at a magnetic flux density $B = 1\text{T}$, by only 14GHz which is equivalent to a change in wavelength of only 0.02nm .

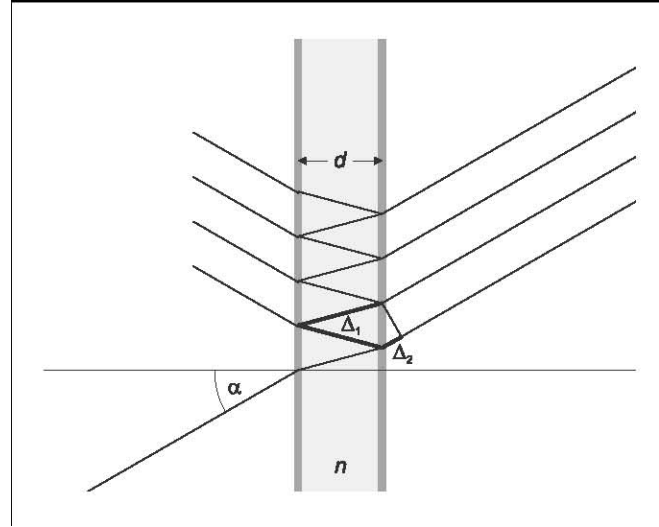


Figure 4. Fabry-Perot etalon as an interference spectrometer. The ray path is drawn for an angle $\alpha > 0$ relative to the optical axis. The optical path difference between two adjacent emerging rays is $\Delta = n(\Delta_1 - \Delta_2)$

In this experiment a Fabry-Perot etalon is used to observe these small changes in wavelength. A Fabry-Perot etalon is a glass plate where the two sides are parallel to a very high precision and with both sides aluminized to make them partially reflective. The slightly divergent light enters the etalon, which is aligned perpendicularly to the optical axis, and is reflected back and forth several times, whereby part of it emerges each time (see Fig. 4). Due to the aluminizing this emerging part is small, i.e., many emerging rays can interfere. Behind the etalon the emerging rays are focused by a lens and a concentric circular fringe pattern associated with a particular wavelength λ can be observed with an ocular (eye-piece).

The rays emerging at an angle of α_k interfere constructively with each other when the path difference between the rays is equal to a whole number of wavelengths (see Fig. 4)

$$\Delta = \Delta_1 - \Delta_2 = 2d\sqrt{n^2 - \sin^2 \alpha_k} = k\lambda \quad (8)$$

Δ = optical path difference, d = thickness of the etalon, n = refractive index of the glass, k = order of interference.

A change in the wavelength of $\delta\lambda$ is seen as a change in the angle, α , of the emerging ray of $\delta\alpha$. Depending on the focal length of the lens, the angle, α , corresponds to a radius, r , and the change in the angle $\delta\alpha$ to a change in the radius δr . If a spectral line contains several components with a difference in wavelength of $\delta\lambda$, each circular interference fringe is split into as many components with the radial distance δr . So a spectral line doublet is recognized by a doublet structure and a spectral line triplet by a triplet structure in the circular fringe pattern.

Part 1. Qualitative observation of the normal Zeeman effect

Setup

The complete experimental setup in transverse configuration is illustrated in Fig. 5.

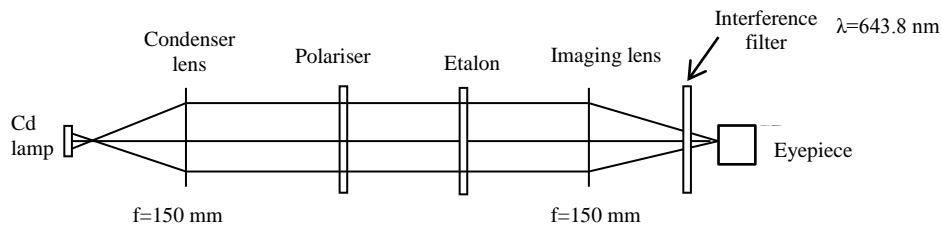


Figure 5. Experimental setup for observing the Zeeman effect in transverse configuration. The position of the left edge of the optics riders is given in cm.

Mechanical and optical setup:

- The cadmium lamp and magnet should be mounted on the rail when you arrive. Be very careful with moving the magnet assembly as the cadmium lamp is very fragile (and expensive!)
- Mount the optical components according to Fig. 5.

Electrical connection:

- Connect the cadmium lamp to the power supply; after switching on wait 5 min until the light emission is bright blue.
- Connect the coils of the electromagnet in series and then to the high current power supply.
- The Hall probe for measuring the magnetic field must NEVER be placed in the magnet when the lamp is on.
- Only increase the current flowing through the magnet when it is being used. The magnet rapidly heats up when in use so always turn the current back to zero when not observing the fringe pattern.

Adjusting the observing optics:

The optimum setup is achieved when the red circular fringe pattern is bright with strong contrast and centred on the line graduation. While adjusting remove the polarization filter so that the observed image is as bright as possible.

- Focus the ocular at the line graduation.
- Move the imaging lens until you observe a sharply defined image of the circular fringe pattern.
- Move the condenser lens until the observed image is illuminated as uniformly as possible.
- Shift the centre of the circular fringe pattern to the middle of the line graduation by slightly tipping the Fabry-Perot etalon with the adjusting screws.

If the adjustment range does is not sufficient then gently rotate the Fabry-Perot etalon with its frame or adjust the height of the imaging lens and the ocular relative to each other.

Carrying out the experiment

a) Observing in transverse configuration:

- First observe the circular fringe pattern without magnetic field ($I = 0\text{ A}$).
- Slowly enhance the magnet current until the fringes split and are clearly separated.
- Record what you observe.

To distinguish between π and σ components:

- Introduce the polarization filter into the ray path (see Fig. 5) and set it to 90° where the two outer components of the triplet structure should disappear.

- Set the polarization filter to 0° and the (unshifted) component in the middle should disappear.

b) Observing in longitudinal configuration:

- Carefully rotate the entire setup of the cadmium lamp with the magnet by 90° .
- First observe the circular fringe pattern without magnetic field ($I = 0$ A).
- Slowly increase the magnet current until the split fringes and are clearly separated.
- Record what you observe.

To distinguish between σ^+ and σ^- components:

- Introduce a quarter-wavelength plate into the ray path between the cadmium lamp and the polarization filter and set it to 0° .
- Set the polarization filter to $+45^\circ$ and -45° . In each case one of the two doublet components should disappear.

Part 2. Quantitative measurements of the Zeeman effect

To perform quantitative measurements of the splitting of the red cadmium spectral line as a function of magnetic flux density the eyepiece, used for the first part of the experiment, is replaced with the linear CCD array and the intensity of a line through the ring system of the interferometer is displayed on the PC. To perform quantitative measurements the lamp is placed in the transverse configuration. The basic configuration should look like the schematic diagram shown in figure 6. Make sure the interference filter is as close as possible to the CCD array so as to stop room light hitting the array.

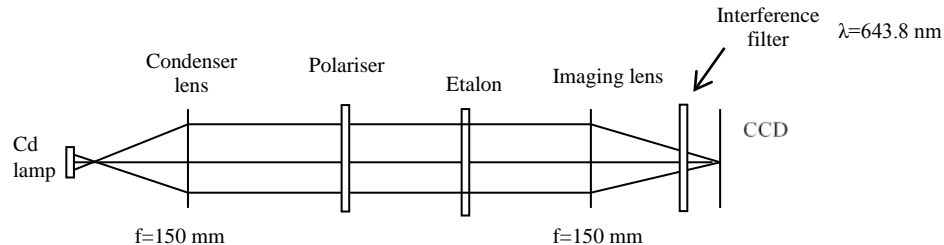


Figure 6. Schematic diagram of optical setup for quantitative measurements

Adjusting the setup

The software for the CCD camera can be used to adjust the camera settings to optimise data collection. For the initial setting up of the experiment set the camera to use only 256 pixels (button with a camera and I in the corner). This will allow the screen to be refreshed more quickly which aids in setting up the experiment. Once the setup is complete set the camera to 2048 pixels (button with a camera and II in the corner). It is also possible to adjust the exposure time (using the buttons with a magnifying glass) so that the peaks have an intensity of approximately 50 %.

To ensure that the CCD is in the focal plane of the imaging lens ($f = 150$ mm), move the imaging lens along the optical axis until the peaks of the observed curve are sharply imaged and show the maximum intensity. The centre of the ring system must then be imaged on the CCD line. For this, you can either move the VideoCom perpendicular to the optical axis or tilt the etalon interferometer slightly using the adjusting screws. The centre of the ring system is on the CCD line when further adjustment does not cause any more peaks to emerge and the two central peaks (left and right intersections of the innermost rings) are the maximum distance apart.

Move the condenser lens until you obtain uniform illumination of the entire CCD line.

Diffraction Angle Calibration

Before you can determine the change in the wavelength of the lines, the individual pixels of the CCD must be calibrated to diffraction angle, α . For an imaging lens of effective focal length, f , the diffraction angle can be calculated using $\alpha = \arctan(x/f)$ where $x = (1024 - p) * 14 \mu\text{m}$ where p = pixel coordinate on the CCD (0 to 2047). This is performed automatically in the software by entering the value for the focal length of the imaging lens used ($f = 150\text{mm}$) into the set up screen. Ask the demonstrators for help with this part of the software if necessary. This calibration changes the horizontal axis to degrees.

It is now necessary to set the zero position to the centre of the ring pattern. Use the zoom and display coordinates functions to determine the angles of the two central peaks and use the mean of these as the zero point shift. Enter the negative value of this into the “shift zero point” box. This should set the centre of the ring system to 0° on the angular scale.

Quantitative evaluation

The aim of this part of the experiment is to measure how much the peaks shift with an applied magnetic field. The angle of the peak without an applied field is α_0 and you can record a series of values of the peak position with increasing magnetic field, α_1 .

The intensity of the peaks should be around 50 % (adjust this if necessary as the luminance of the Cd lamp changes in the magnetic field). Use the zoom and display coordinates function to determine the centre of one of the peaks with the magnetic flux density at 0T (α_0). Record the position of one of the peaks (α_1) as the magnet current is increased (use steps of 0.5A in the current through the magnet). The value of the magnetic flux density can be found for each value of magnet current at the end of the experiment. When you have finished all the measurements and the lamp is switched off place the Hall probe carefully into the centre of the magnet (It is the small square which is the sensor and this should be visible through the hole in the magnet pole pieces). Then record the magnetic flux density as a function of magnet current.

Repeat the measurements in the longitudinal configuration using the quarter-wave plate and polariser.

You should now have two tables of angular position of the peak, α_1 , as a function of magnetic flux density, B . The aim is to convert this into a shift in the energy of the peak so that equation 5 can be used to measure the value of the Bohr magneton, μ_B . If one knew the interference order, k , for the peak you are measuring then knowing the angular position of the peak and the thickness and refractive index of the etalon, one would be able to determine the wavelength of the peak from equation 8. Unfortunately, it is not possible to know the interference order for any particular peak. However, each peak has a single value of k and as the peak moves with magnetic flux density this is due to the change in wavelength with k remaining constant. It is therefore possible to determine the relative shift in wavelength, $\Delta\lambda/\lambda$, from the shift in the angular position of a given peak.

If the angular position of the peak at zero magnetic field is α_0 , then the relative shift in wavelength is given by,

$$\frac{\Delta\lambda}{\lambda} = \frac{\cos \beta_1}{\cos \beta_0} - 1 \quad (9)$$

where

$$n = 1.46 = \frac{\sin \alpha}{\sin \beta} \quad (10)$$

where α is the external angle a ray makes with the etalon (what you are measuring) and β is the internal angle the same ray makes inside the etalon. Equation 9 is derived from equation 8, and equation 10 is Snell's Law.

Using these equations calculate $\Delta\lambda/\lambda$ as a function of magnetic flux density for your data. Finally you need to convert $\Delta\lambda/\lambda$ to ΔE . To do this use equation 11.

$$\Delta E = -\frac{\Delta\lambda}{\lambda} E = -hc \frac{\Delta\lambda}{\lambda^2} \quad (11)$$

where $\lambda=643.8$ nm for the cadmium line. Use your data to calculate the value of the Bohr magnetron, μ_B .

The literature value is $57.9 \mu\text{eV/T}$.

Derivation of equation 9

Any given peak has a value of k and as λ changes due to the magnetic field k remains constant, Therefore,

$$\frac{\Delta\lambda}{\lambda} = \frac{2d\sqrt{n^2 - \sin^2 \alpha_1} - 2d\sqrt{n^2 - \sin^2 \alpha_0}}{2d\sqrt{n^2 \sin^2 \alpha_0}} \quad (12)$$

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\left(\frac{n^2 - \sin^2 \alpha_1}{n^2 - \sin^2 \alpha_0}\right)} - 1 \quad (13)$$

From Snell's law

$$\sin \alpha = n \sin \beta \quad (14)$$

$$n^2 \sin^2 \alpha = n^2 (1 - \sin^2 \beta) = n^2 \cos^2 \beta \quad (15)$$

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\left(\frac{n^2 \cos^2 \beta_1}{n^2 \cos^2 \beta_0}\right)} - 1 = \frac{\cos \beta_1}{\cos \beta_0} - 1 \quad (16)$$