Nuclear Physics and Astrophysics

PHY-302

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Lecture 11 - Beta Decay

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Beta Decay

β decay played important role in understanding nuclear properties

First hint of new force of nature & new form of matter (neutrino ν_e)

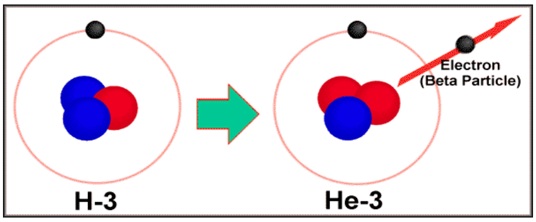
Gave rise to understanding of particle symmetries

In particular: violation of parity inversion!

Early studies of radioactive nuclei showed electron emission from nuclei

Positron (anti-electron) emission from nuclei first observed in 1934

Electrons & positrons do not exist in nuclei...?!?!



(A,Z) → (Z+1,A) + e^- or (A,Z) → (Z-1,A) + e^+

$$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z+1}\mathrm{Y}_{N-1} + e^{-} + \bar{\nu}_{e} \quad \text{or} \quad {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z-1}\mathrm{Y}_{N+1} + e^{+} + \nu_{e} $$

A is conserved, but not Z or N !!!

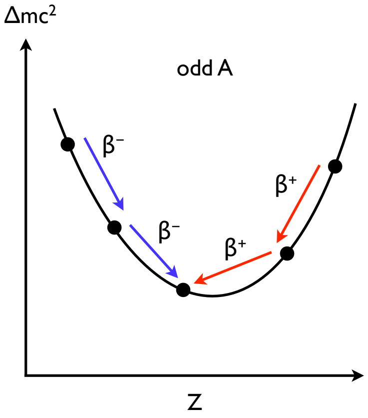
Process is interpreted as nucleon conversion

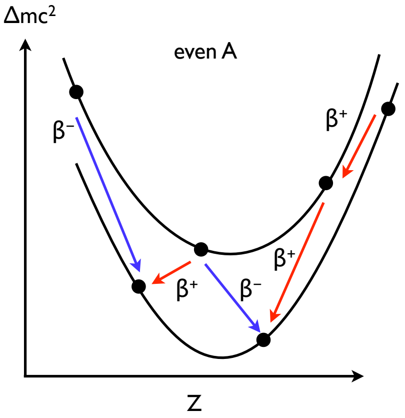
For constant A, beta-decay nuclei achieve stability by converting n to p or p to n

For odd A, only one parabola is observed

For even A two parabolas are observed

- decays switch from one to the other
- due to pairing term in SEMF





For odd A only lowest mass state is β stable

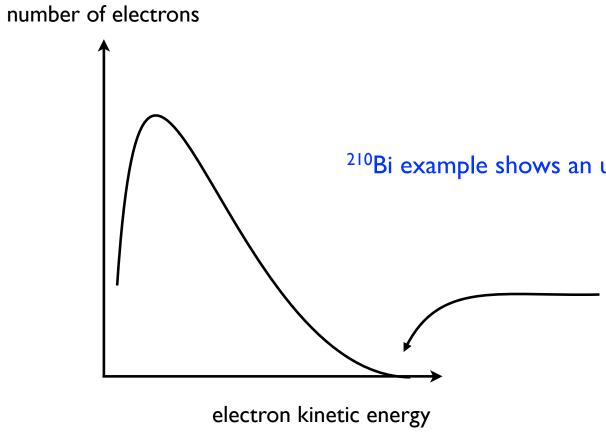
For even A two lowest mass states are β stable

How would energy spectrum of beta-decay electrons look?

Beta decay electrons / positrons have continuous energy spectrum

Expect peaks associated with energy difference in initial & final states

No peaks observed as for alpha-decay!



^{210}Bi example shows an upper energy limit

Value of Max kinetic energy of electron (T)
occurs at T = Q

Initial hypothesis: peaks smeared out by collisions with atomic electrons

- Maximum energy of beta-decay electrons is Q
- In that case energy should appear as heat
- precise experiment could not verify this

1931: Pauli proposed second particle is emitted

- Termed neutrino by Enrico Fermi
- Neutrino carries away missing energy
- It is undetected in experiment
- Charge conservation requires neutrino to be neutral
- Angular momentum arguments imply spin must be ½
- Neutrino is produced in e^+ emission / anti-neutrino in e^- emission

note: 'neutrino' often used to denote both neutrino & anti-neutrino
similarly for electron
important to distinguish in writing decay process equations

In 1938 related process of electron capture was observed:
orbital electron captured by nucleus

$n \rightarrow p + e^- + \bar{\nu}_e$ negative β⁻ decay

$p \rightarrow n + e^+ + \nu_e$ positive β⁺ decay

$p + e^- \rightarrow n + \nu_e$ electron capture

Consider free neutron decay ($t_{1/2} \approx 10$ mins) $n \rightarrow p + e^- + \bar{\nu}_e$

Define Q for reaction in usual way: difference in initial & final masses

$$Q = (m_n - m_p - m_e - m_{\bar{\nu}})c^2$$

For neutrons at rest: $Q = T_p + T_e + T_{\bar{\nu}}$ ignoring p^+ recoil (only 0.3 KeV)

This is <u>measured</u> to be Q=0.782 ± 0.013 MeV

electron & neutrino share the 'missing' energy: max e^- energy = min neutrino energy

calculation of Q using masses and comparison to measurement ⇒ infer neutrino mass

$$m_{\bar{\nu}}c^2 = m_n c^2 - m_p c^2 - m_e c^2 - 0.782 \ \ MeV/c^2$$
$$m_n = 939.573 \ \ MeV/c^2$$
$$m_p = 938.280 \ \ MeV/c^2$$
$$m_e = 0.511 \ \ MeV/c^2$$

thus neutrino can be considered massless

Fermi developed the theory of β decay on neutrino hypothesis

Neutrino hypothesis verified by careful experiment:

measure momentum of nuclear recoil & electron

low energy nuclei are easily scattered - difficult to measure

can be achieved in few cases

applying momentum conservation, data is consistent with unobserved massless particle

Consider: $${}^{A}_{Z}\mathrm{X}_N \rightarrow {}^{A}_{Z+1}\mathrm{Y}_{N-1} + e^- + \bar{\nu}_e$$

Note: kinematics must be relativistically treated

$E_e = T_e + mc^2$

E_e ~ MeV therefore T » mc^2

m are nuclear masses $$Q_{\beta^-} = \left[m_N({}^{A}_{Z}\mathrm{X}) - m_N({}^{A}_{Z+1}\mathrm{Y}) - m_e\right]c^2$$

B^A_i is binding energy of ith electron

convert nuclear to atomic masses: $$m_A({}^{A}_{Z}\mathrm{X})c^2 = m_N({}^{A}_{Z}\mathrm{X}) + Zm_ec^2 - \sum_{i=1}^{Z} B_i^A$$

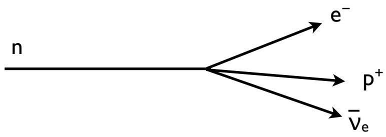
$$Q_{\beta^-} = \left([m_A({}^{A}_{Z}\mathrm{X}) - Zm_e] - [m_A({}^{A}_{Z+1}\mathrm{Y}) - (Z+1)m_e] - m_e\right)c^2 + \left(\sum_{i=1}^{Z} B_i^A - \sum_{i=1}^{Z+1} B_i^A\right)$$

m_A are atomic masses

Ignoring difference in binding energy, then $$Q_{\beta^-} = [m_A({}^{A}_{Z}\mathrm{X}) - m_A({}^{A}_{Z+1}\mathrm{Y})]c^2$$ in atomic masses

For β^+ decay: $$Q_{\beta^+} = [m_A({}^{A}_{Z}\mathrm{X}) - m_A({}^{A}_{Z-1}\mathrm{Y}) - 2m_e]c^2$$

- In ɣ decay nucleus emits a photon
- Similarly Fermi postulated neutron spontaneously creates electron & electron anti-neutrino and converts to proton



Fermi Theory of β decay is consistent with data if coupling of particles has very small value - i.e. very small probability of occurring

We found neutrino was consistent with having zero mass

It turns out this question has two very different implications:

- solar neutrino problem
- mass-energy density of the universe

But for now lets return to beta decay...

- To understand decays in QM framework Schrödinger Eq. is insufficient
  - time independent
- Use technique of Perturbation Theory
- Nuclear potential P = V + V'
- V gives rise to stationary (stable) states
- V' is a weak additional potential leading to transitions between states

Method is to use Schrödinger eq. for potential V
then use these wave solutions to calc. transition probability between states
transition probability is the decay constant λ

Decay const. given by Fermi's Golden Rule

$$\lambda = \frac{2\pi}{\hbar}|V_{fi}|^2\rho(E_f)$$

$V_{fi} = \int \psi_f^* V \psi_i dv$ is known as the matrix element for the decay

Trans. prob. also influenced by the density of final states $\rho(E_f)$
Number of states within energy interval dE_f is $\mathrm{d}n_f = \rho(E_f)\mathrm{d}E_f$

What is the neutrino?

There are three types and associated anti-particles:

neutrino	spin	charge	mass
ν_e	½	0	< 3 eV
ν_μ	½	0	< 0.19 MeV
ν_τ	½	0	< 18.2 MeV

neutrinos also carry a lepton number
+1 for neutrinos
-1 for anti-neutrinos

Beta decay only involves electron type neutrinos

Neutrino was inferred from 'violation' of energy-momentum conservation in beta-decay

Direct observation of neutrino was not made until 1956

Lets look at its interaction cross section for the reaction:

$$\nu_e + p \rightarrow n + e^+$$

$$\sigma = \frac{\text{probability per target atom for reaction to occur}}{\text{incident flux of neutrinos}}$$

Using **Fermi's Golden Rule** we can calculate cross section

Neutrino flux determined using plane wave approximation

$$\sigma = \frac{2\pi}{\hbar c} g^2 |M_{fi}|^2 \frac{4\pi p E}{\hbar^3 c^2}$$

Estimate matrix element M_{fi} numerically from neutron beta decay
Take neutrino energy to be 2.5 MeV (larger than threshold for reaction to occur)

$$\sigma = 1.2 \times 10^{-19} \text{ barns}$$
$$= 1.2 \times 10^{-43} \text{ cm}^2$$

Note: cross section for nucleon-nucleon scattering ~ 20 barns!

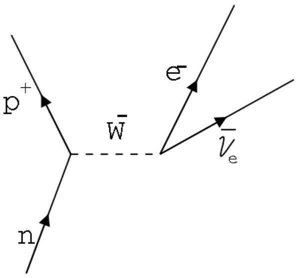
Attempt to measure this cross section by increasing probability:
given cross section is for one nucleon
measure interaction with many nucleons

Typical solid has ~10^{24} protons per cm^3

Neutrino reaction cross section is 10^{-43} cm^2 per proton
Thus net probability per centimeter is 10^{-19} !!!

For probability ~1 we require 10^{19}cm of material
10 light years!!!

The neutrino only interacts via a new force of nature called the "weak" force (obviously!)
→ it is responsible for all beta decay processes and all neutrino interactions



beta decays caused by
emission of a heavy W particle
Large mass accounts for short
range of weak force