Nuclear Physics and Astrophysics

PHY-302

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Lecture 10 - Alpha Decay

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Material For This Lecture

New topic!

- Alpha decay:
- Definition
- Quantum mechanics of tunnelling process
- Application to alpha decay
- Comparison of model to experimental data

Nuclear reaction equation for alpha decay

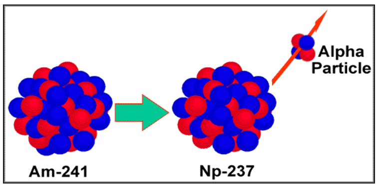
$$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A-4}_{Z-2}\,\mathrm{X}_{N-2} + \alpha $$

X = parent (or mother) nucleus Y = daughter nucleus

Alpha decay is <u>spontaneous</u> emission of ${}^{4}_{2}$He from an unstable nucleus

“spontaneous” = kinetic energy appeared without cause

Can be understood as simple quantum mechanical process: **Tunneling**



Alpha decay is calculable

Will provide information about nuclear structure

Least penetrating of all emissions - stopped by ~ few centimetres air

charge = +2 ⇒ strongly ionising, loses energy quickly in Coulomb scatters

Alpha emission: Coulomb repulsion is important for large A

Coulomb repulsion ~ Z^2

Nuclear attraction ~ A

Heavy nuclei are observed to emit alpha particles

appearance of K.E. comes from mass:

total mass is reduced ⇒ total Binding energy is increased

α particle very stable - tightly bound ⇒ large B , small mass

Let's see why...

$$ {}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + \alpha $$

Energy-momentum conservation for X at rest gives:

$$ M_X c^2 = M_Y c^2 + T_Y + M_\alpha c^2 + T_\alpha $$

T=kinetic energy

Recall*: calculate Q value for reaction:

$$ Q = (\sum_i M_i - \sum_f M_f)c^2 $$
$$ = T_f - T_i $$

T_i=0 and T_f = T of alpha (assuming Y doesn't recoil)

Q value is net energy release - can write in terms of Binding Energy, B

Spontaneous decay ONLY if Q>0

$$ Q = (M_X - M_Y - M_\alpha)c^2 $$
$$ = B_Y + B_\alpha - B_X $$
$$ B_\alpha > B_X - B_Y $$

Q = total kinetic energy (assuming decay at rest)

thus Q is maximised by having B_α and B_Y large
i.e. m_α and m_Y small

* discussed in lecture 5

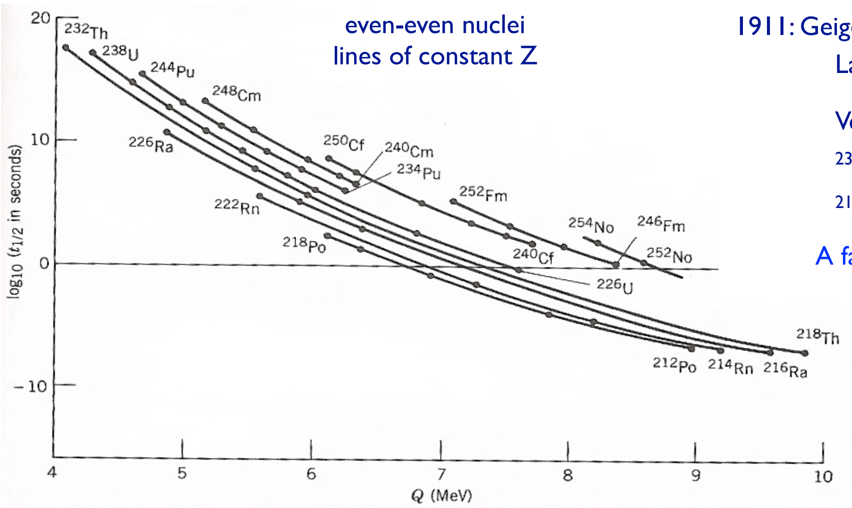


Figure 8.1 The inverse relationship between α-decay half-life and decay energy, called the Geiger-Nuttall rule. Only even-Z, even-N nuclei are shown. The solid lines connect the data points.

even-even nuclei
lines of constant Z

1911: Geiger & Nuttall

Large Q for α emitters ⇒ small $t_{1/2}$

Very Strong effect:

^{232}Th: $t_{1/2}$ = 10^{10} y Q = 4.08 MeV

^{218}Th: $t_{1/2}$ = 10^{-7} s Q = 9.85 MeV

A factor 10^{24} in $t_{1/2}$ for factor 2 increase in Q!!

Large Q means large diff in B between X and Y nuclei
Means X more unstable
Decay more likely to occur

note: ^{235}U has long $t_{1/2}$
If it had shorter $t_{1/2}$ by factor 1000, there would be no naturally occurring U - no nuclear industry

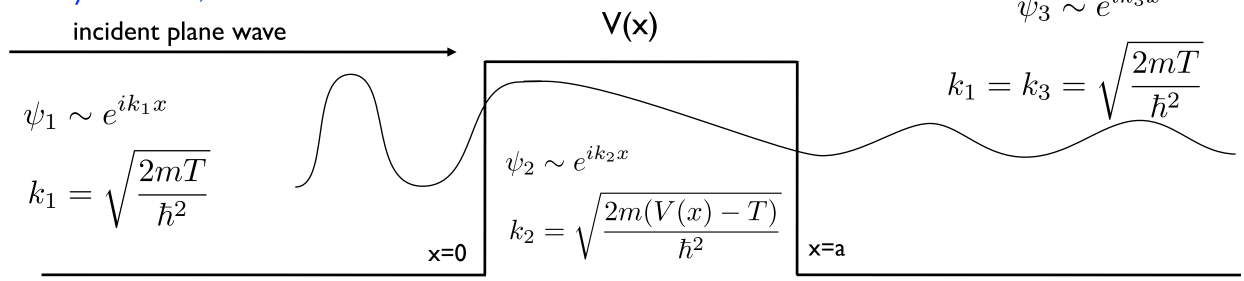
Theory of α Emission

1928: Attempt to understand α emission in QM framework

- α particle is preformed daughter moving within potential of nucleus
- little evidence to believe pre-formation
- this approach if vindicated means α particle behaves as if it is preformed

Quantum mechanical approach: tunnelling of α particle from nuclear / Coulomb potential

* useful for you to know, but mathematical solutions will not be examined



To solve a potential barrier problem in general:

p = momentum
T = kinetic energy
m = mass

$$p = \hbar k = \sqrt{2mT}$$

- solve Schrödinger equation in 3 regions for particle $E < V_0$

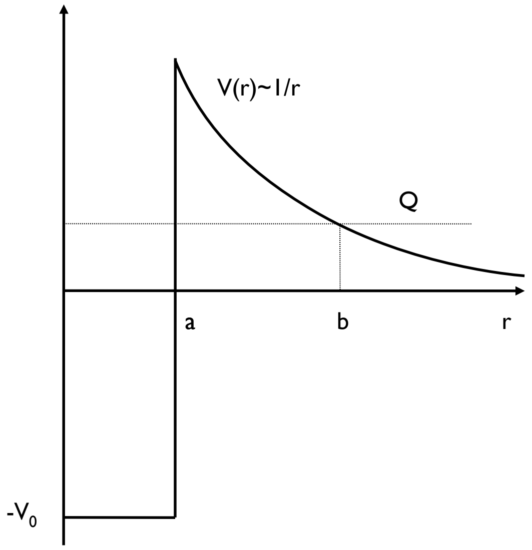
x<0	V=0
0<x<a	$V=V_0$
x>a	V=0

Transmission probability $\sim e^{-2k_2 x}$

- apply boundary conditions at x=a and x=0:
  Ψ must be continuous across a boundary
  derivative dψ/dr must also be continuous across boundary

- Classically particle is forbidden in region 0<x<a, and is not observed in x>a
  particle keeps bouncing off potential barrier wall until it finally penetrates

- Particle crosses barrier with reduced amplitude & same T !



- 2p and 2n fuse to form α
- Nuclear potential : square well, width a
- For r>a only Coulomb potential operates
- For a<r<b potential V > Q (classically particle is bound)
- For r > b particle is classically allowed
- Quantum mechanically wavefunction leaks into forbidden region
- α emission rate depends on tunnelling probability
- Explains why α emitters do not decay immediately
- BE of ^{4}He is 28 MeV
- Q ~ 5 MeV & V_0 ~ 35 MeV (well depth)
- Barrier height at r=a is ~ 34 MeV

For ^{235}U leakage prob is very low ($t_{1/2}$ very large), α particle hits barrier ~10^{38} times before penetration!

In 'semi-classical' approach: decay constant, λ = f P

f = frequency of collisions with barrier

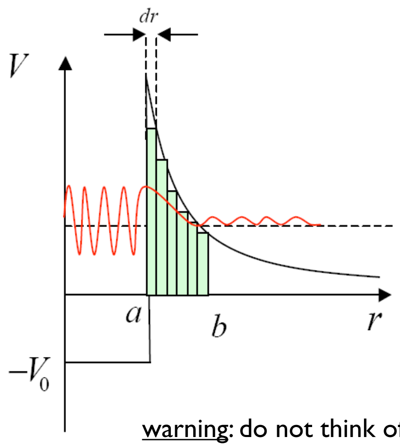
P = probability of transmission through barrier

recall: $t_{1/2} = \dfrac{\ln 2}{\lambda}$

Estimate f ~ v/a with v given by kinetic energy of α particle in daughter nucleus (~Q)

For 8 MeV v~10^7 m/s i.e. f~$10^7/10^{-14}$m = 10^{21} /s if P~10^{-29} then λ~10^{-8}

Exact solution very similar to method described



Can consider Coulomb barrier as series of infinitesimal rectangular barriers with height V(r), width dr

Then probability for penetration of each infinitesimal barrier is dP:

$$\mathrm{d}P = exp\left(-2\mathrm{d}r\sqrt{\frac{2m}{\hbar^2}[V(r)-Q]}\right)$$

Probability to penetrate barrier is $P = e^{-2G}$ Larger P for larger Q!

G is Gamov factor: $G = \sqrt{\dfrac{2m}{\hbar^2}}\displaystyle\int_a^b \sqrt{V(r)-Q}\,\mathrm{d}r$

warning: do not think of the particle moving from r=a to r=b in 'forbidden region'
we are calculating a wave function - particle exists in all places with different probability!

You cannot imagine the alpha particle sitting in this region and experiencing a repulsive Coulomb force

Why does Uranium not decay via Mg emission?

In low momentum approximation (low kinetic energy approximation):

$$Q = \frac{1}{2}mv^2 \ll \frac{Zze^2}{4\pi\epsilon_0 R}$$

R is nuclear radius ($R \sim R_0 A^{1/3}$)

Particle energy much less than maximum barrier height

Gamov factor simplifies:

$$G \simeq \sqrt{m}\frac{Zze^2}{4\pi\epsilon_0 \hbar v}$$

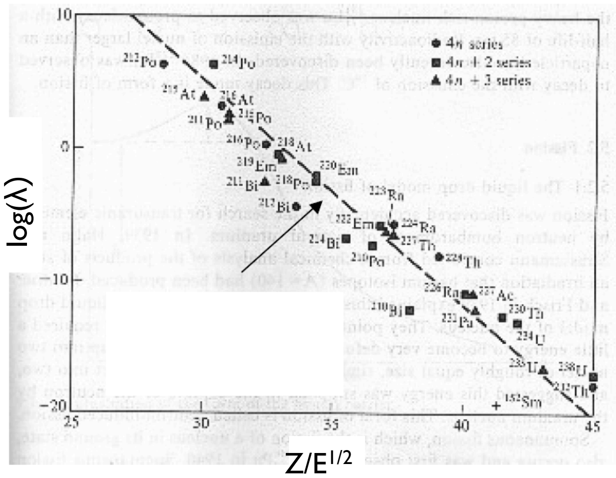
v is velocity of outgoing particle

or in terms of α particle kinetic energy T (= Q)

$$G \propto Z\sqrt{\frac{m}{Q}}$$

$$\lambda = fP$$
$$= fe^{-2G}$$

$$\log\lambda = 57 - 1.7Z\sqrt{\frac{m}{Q}}$$



Approximation holds for most heavy nuclei

We made lots of approximations and neglected many factors!

- probability of pre-forming α-particle in nucleus
- ignored Fermi's Golden Rule (lecture 3)

$$\boxed{\lambda = \frac{2\pi}{\hbar}|V_{fi}|^2\rho(E_f)}$$

- (should calc wavefunc. for complete nucleus, and prob. of finding it in α + X' state)
- this probability should depend on whether nucleus is ee/oe/oo
- ignored angular momentum considerations
- shape of potential used is idealised case
- assumed nuclei are spherical, α emitters have large A, nuclei are often deformed
- Calculation strongly depends on value of R used
- 4% change in R leads to change in $t_{1/2}$ by factor 5!

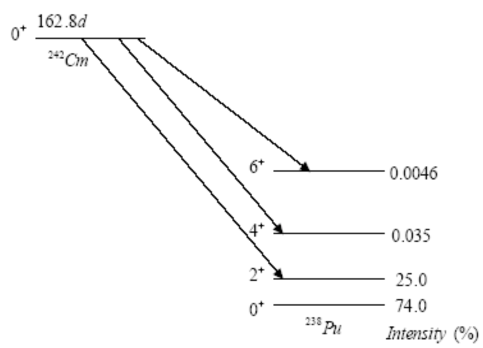
Despite this, calculation is accurate to ~ factor 50 over 20 orders of magnitude

Isotopes of Thorium

A	Q(MeV)	Half Lives $t_{1/2}$ (s) Measured	Calculated
220	8.95	10^{-5}	$3.3x10^{-7}$
222	8.13	$2.8x10^{-3}$	$6.3x10^{-5}$
224	7.31	1.04	$3.3x10^{-2}$
226	6.45	1854	$6.0x10^{1}$
228	5.52	$6.0x10^{7}$	$2.4x10^{6}$
230	4.77	$2.5x10^{12}$	$1.0x10^{11}$
232	4.08	$4.4x10^{17}$	$2.6x10^{16}$

Expected and measured half-lives agree to within factor ~50

Energy level diagram for Alpha decay of ^{242}Cm to 4 states of ^{238}Pu

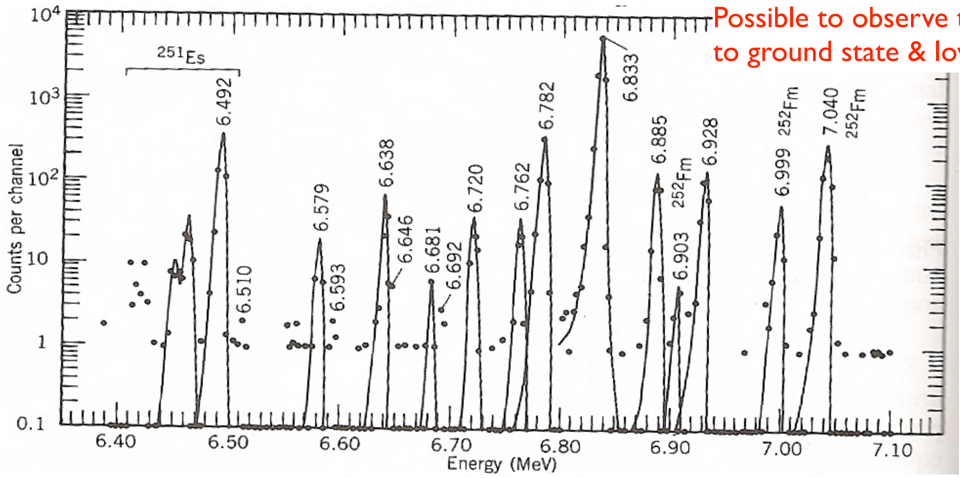


- Alpha emission does not leave daughter nucleus in ground state
- Intensity is fraction of all alpha-decays to that state
- Each has different Q value:
  Q value to ground state - excitation energy
- Intensity reduces as excitation energy increases

spectroscopy of alpha-decay can reveal energy levels of excitations
measurement gives kinetic energy of α particle ⇒ determine Q value

Assuming highest energy α decay is to ground state
Each state will (usually) decay to ground state or lower excitation via γ-ray photon emission
observe α and γ emission in co-incidence gives confidence in determination of energy levels

spectrum of ^{251}Fm alpha decays to ^{247}Cf



Possible to observe transition to ground state & lower excited states

More about spectroscopy later