Nuclear Physics and Astrophysics

PHY-302

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Lecture 8 - Nuclear Shell Structure

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Material For This Lecture

We will look at nuclear properties and notice familiar patterns

Interpret this as shell-like structure of nucleons (much like atomic electrons!)

Learn shell structure can be "derived" from a finite nuclear potential

We could try to apply calc's of deuteron & NN scattering to heavier nuclei

Complex many body problem

Experiments suggest nuclear force has additional correlations - more complexity!

Study a simplified model using physics insight

Shell model initially used for Atomic Structure

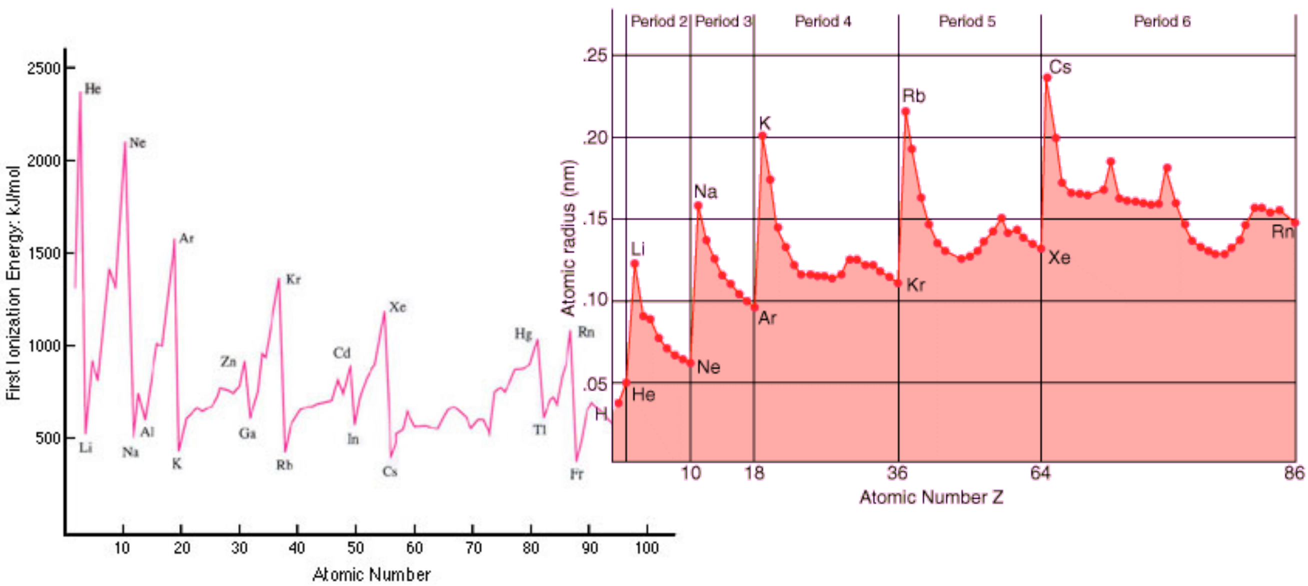
Atomic model:

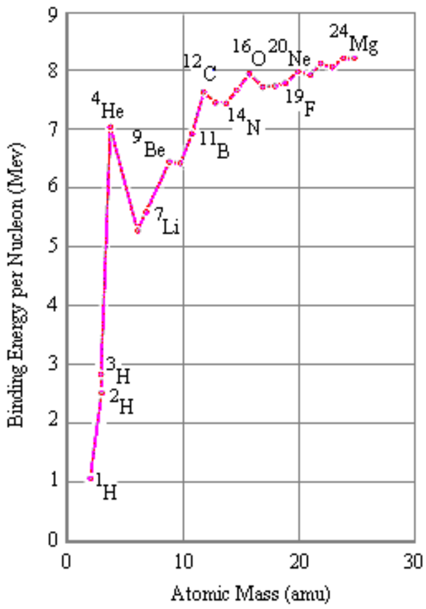
- e^- are filled in increasing energy shells
- impose Pauli Exclusion principle for identical fermions
- atomic properties determined by outermost shell - valence e^-
- inner shell (closed) shells are 'inert'
- potential due to nuclear Coulomb field
- Schrödinger equation can be solved for potential

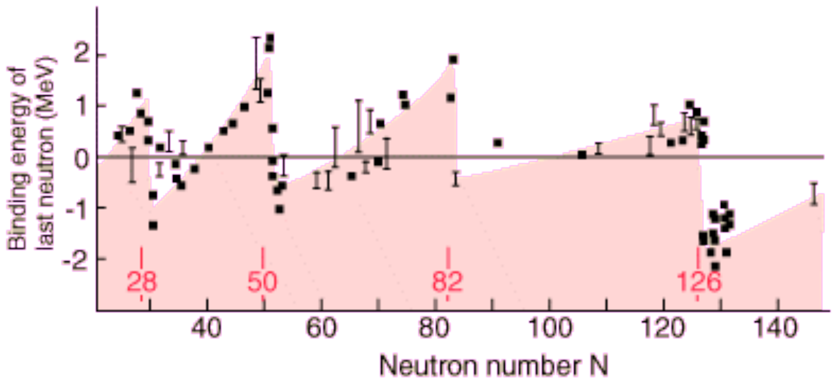
examine <u>atomic</u> properties across periodic table

smooth transitions within shell

sharp discontinuities across shell boundary



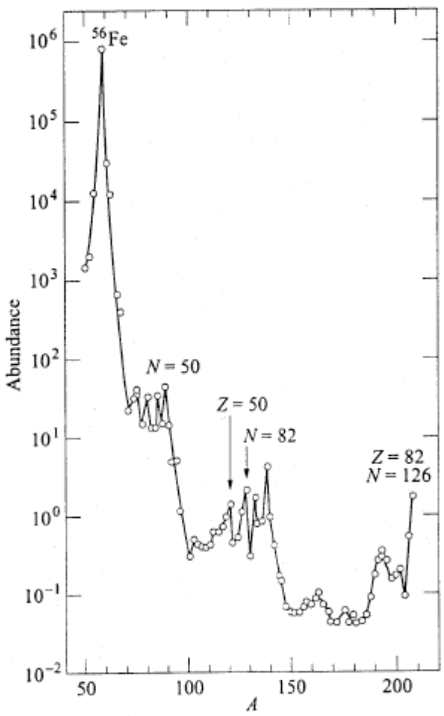




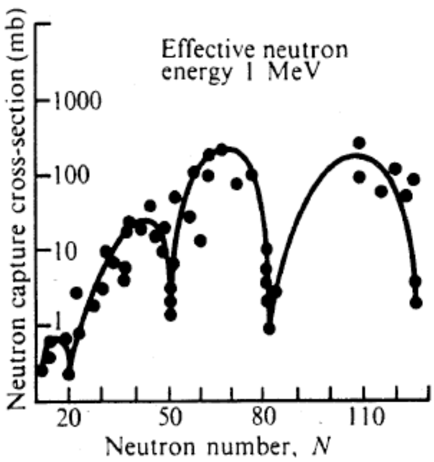
Look at B/A curve at low Z

Also BE of last neutron vs N relative to SEMF prediction

Spikes appear at specific values of N



relative abundance of elements shows details
spikes occur when N/Z have specific values



cross section for n absorbtion vs N
For certain N cross section drops by factor ~100

These same magic numbers occur repeatedly

Sharp discontinuities occur when N or Z are:

2, 8, 20, 28, 50, 82, 126

Nuclei with N,Z = 'magic number' have higher BE

Similar to closed atomic shell

Shell Model interprets data in this way

Successfully describes these phenomena

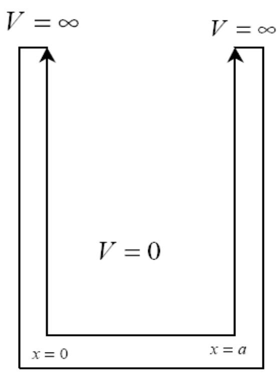
Basic assumption of Shell Model:

- each nucleon moves in potential: average of all other nucleons
- net effect of nuclear motions makes potential vary smoothly
- each nucleon is bound ⇒ potential is a potential well
- each nucleon moves in 'orbit' of that potential well

Infinite square well in 1 dimension - a quick reminder

Particle trapped between x=0 and x=a

walls are impenetrable ⇒ ψ = 0 for x<0 and x > a



$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

$$V(x) = \inf \quad x<0 \;\; ,x>a$$
$$= 0 \quad 0 \leq x \leq a$$

Solve Schrödinger equation for this V(x)

Wave function solution is:

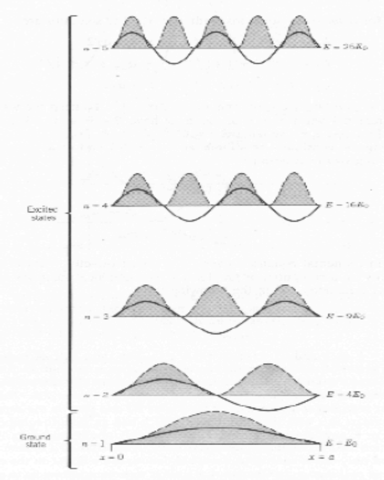
$$\psi = A\sin kx + B\cos kx$$

Imposing boundary conditions quantises energy states

$$E_n = \frac{\hbar^2\pi^2}{2ma^2}n^2 \quad n = 1, 2, 3...$$

Each bound state of a potential well has a unique energy
These are eigenstates of the system

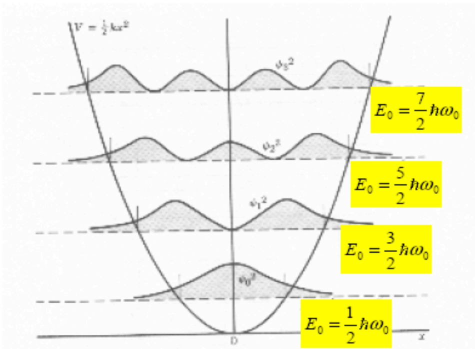
Infinite Potential Well



Simple Harmonic Oscillator

$V \propto x^2$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\frac{1}{2}kx^2\psi(x)=E\psi(x)$$



<u>Energy of any state uniquely specified by quantum number n</u>

Now how about in 3 dimensions?
Same game applies: pick a potential and solve 3d Schrödinger equation

Choosing V = V(r) only ... i.e. not V(r,θ,Φ) then solutions simplify:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(r,\theta,\phi)}{\partial r^2}+V(r)\cdot\psi(r,\theta,\phi)=E\cdot\psi(r)\cdot Y(\theta,\phi)$$

Y(θ,φ) is part of wavefunction with quantised angular momentum

Energy quantisation leads to labelling of states with quantum number *n*

Angular momentum is quantised with states labelled as
- *l* for orbital ang. mom.
- *s* for spin ang. mom.
- *j* for total ang. mom.

For orbital angular momentum often use spectroscopic notation

l =	0	1	2	3	4	5
symbol	s	p	d	f	g	h

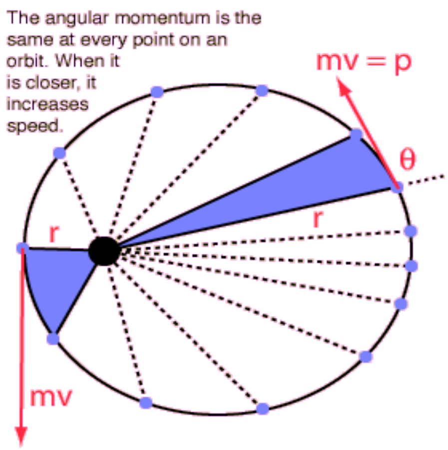
An *l* = 4 state is labelled a g state
Note!!! *s* is a quantum number for spin, but s is a symbol for orbital ang. mom=0

Angular momentum is classically conserved:

L = r x p (cross product of radius and linear momentum p)

In quantum systems:

- composed of orbital motion and intrinsic spin
- quantised in units of ħ
- spin intrinsic property of the particle! (often denoted s)
- s=½ for p, n, e
- behaves like angular momentum
- particles also have ang. mom. due to orbital motions
- particle spin and orbital angular momentum combine
- gives total angular momentum of the system
- this is quantised - plays a large role in nuclear structure - beyond our scope!



Total angular momentum of a nucleus: J
consists of orbital and spin ang mom

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

vector sum of momenta

We label different angular momentum states by the quantum numbers *l* and *j*

As a reasonable guess for nuclear potential take 3D potential well

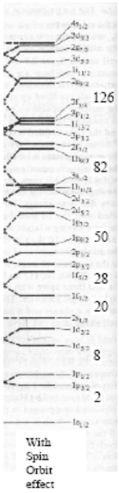
detailed solutions not required - too involved for our purposes
(details in Krane p27-33 and Williams 8.3-8.5)

- Essential result is energy E depends on *n, l, j* quantum numbers
- use spectroscopic notation for quantum number *l*
- Energy levels can occupy different ang. mom. states
- There are (2*j*+1) degenerate states exist
- Final classification of states is written nl_j
- Consider state labelled as $1p_{3/2}$

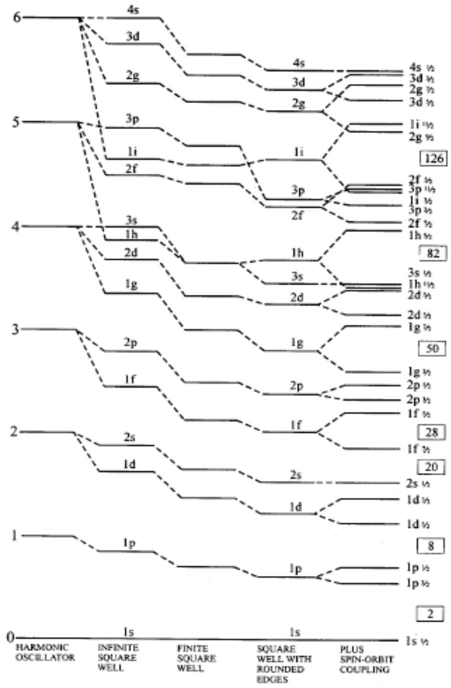
n= 1
l= 1
j= 3/2   Total degeneracy = (2*j*+1) = 4

Solving the Schrödinger equation for a finite potential well and taking into account angular momentum yields the energy levels of the nucleus!

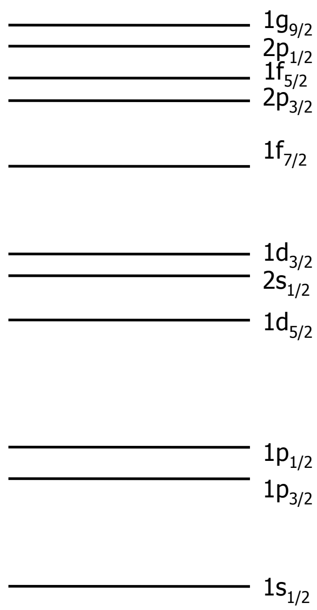
Now we can reproduce the observed magic numbers



Max number in Shell	Total Angular Momentum Levels in Shell
126	$1h_{9/2}, 2f_{1/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2}$
82	$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$
50	$2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$
28	$1f_{7/2}$
20	$1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$
8	$1p_{3/2}$, $1p_{1/2}$
2	$1s_{1/2}$



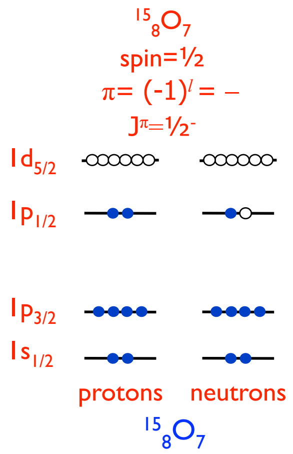
Quantised Energy levels

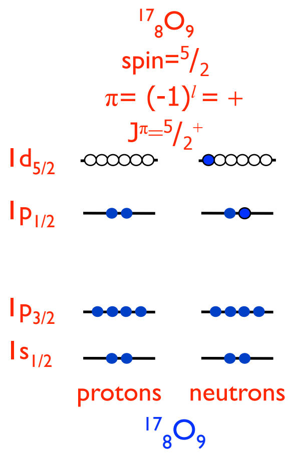


Independent Particle Model

Extension to shell model predicts total nuclear spin given by last unpaired nucleon's angular momentum only (odd-even nuclei only)

Parity π is given by $(-1)^l$





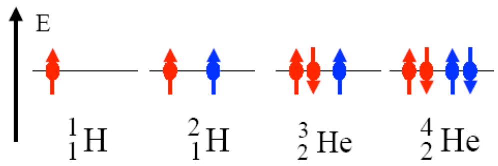
8 protons fill a shell
7^{th} neutron in $p_{1/2}$ shell (j=1/2 l=1)
Expect $^{15}_{8}O_7$ to have
total nuclear spin = ½ (j for last nucleon)
odd parity $P=(-1)^l$

8 protons fill a shell
9^{th} neutron in $d_{5/2}$ shell (j = 5/2 l=2)
Expect $^{17}_{8}O_9$ to have
nuclear spin = 5/2 (j for last nucleon)
even parity $P=(-1)^l$

experiment confirms these predictions
works for almost complete A range
triumph of shell model

How can nucleon occupy such orbits without collision with another nucleon?

Pauli Exclusion Principle saves us...



As nucleons are added they enter lowest energy state

neutrons & protons are distinguishable

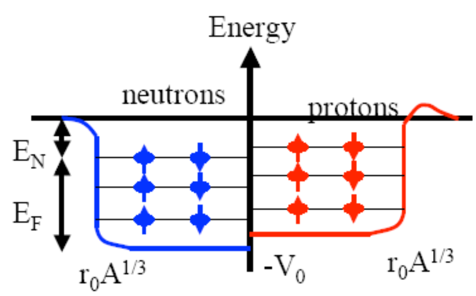
⇒ have own potential wells

restricted by exclusion principle

neutrons / protons are added with opposite spins

collisions can only promote nucleon to next free shell

lower energy collisions forbidden!



Implication of this structure:

nuclei can have excited states!

just like promotion of atomic electron to higher orbital

consider ^{12}C: 6 protons, 6 neutrons

a collision could 'knock' nucleon from ground state to higher energy state

$$^{12}C \rightarrow {}^{12}C^*$$

the * indicates an excited state

$^{12}C^*$ will decay back to ground state (usually by photon emission)