Nuclear Physics and Astrophysics

PHY-302

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Lecture 7 - The Semi-Empirical Mass Formula

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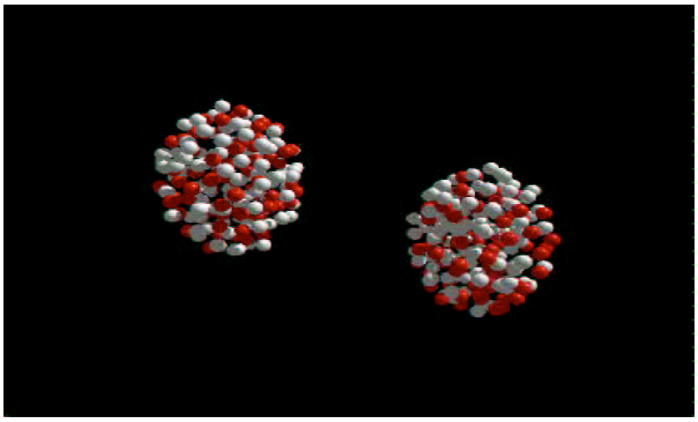
Material For This Lecture

Today we will cover Liquid Drop Model

- Motivation
- Model terms and Parameters
- Applications:
  - Nuclear Masses
  - Binding Energies
  - Conditions of Stability - Introduction
  - Implications of Nuclear Decay
  - Applications For Nuclear Reactions



The liquid drop model:

earliest model of nucleus

nucleus thought of as collection of bound objects

'objects' are in constant motion

- Many other models exist describing nuclear phenomena
- Liquid Drop Model gives us the quantitative Semi-Empirical Mass Formula
- Quantifies properties of nuclei:

    binding energies
    mass
    stability
    decays

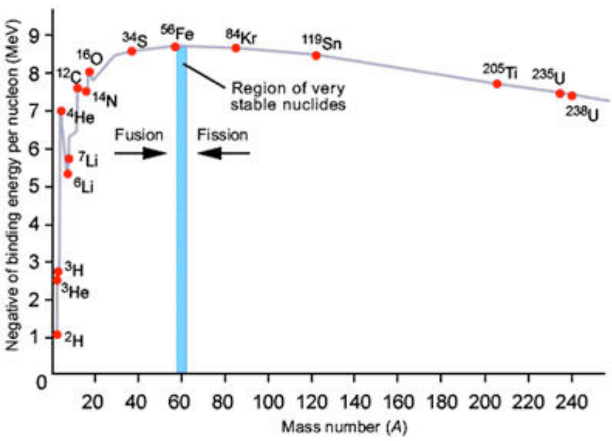
- Not a fundamental model - semi-empirical
- Only has qualitative treatment of nuclear force
- Quantitative power comes from fitting model parameters to data
- Nonetheless its makes some powerful quantitative predictions

how do we measure binding energy?

measure nuclear mass and sum of nucleons to determine B

Element		Mass of Nucleons (u)	Nuclear Mass (u)	Binding Energy (MeV)	Binding Energy MeV/Nucleon
Deuterium	D	2.01594	2.01355	2.23	1.12
Helium 4	4He	4.03188	4.00151	28.29	7.07
Lithium 7	7Li	7.05649	7.01336	40.15	5.74
Beryllium 9	9Be	9.07243	9.00999	58.13	6.46
Iron 56	56Fe	56.44913	55.92069	492.24	8.79
Silver 107	107Ag	107.86187	106.87934	915.23	8.55
Iodine 127	127I	128.02684	126.87544	1072.53	8.45
Lead 206	206Pb	207.67109	205.92952	1622.27	7.88
Polonium 210	210Po	211.70297	209.93683	1645.16	7.83
Uranium 235	235U	236.90849	234.99351	1783.8	7.59
Uranium 238	238U	239.93448	238.00037	1801.63	7.57

A



This plot has important implications
Explains abundance of Iron
Offers understanding of nucleosynthesis
Will study this in greater detail...

Liquid drop model arose from observation that B/A ~ constant across periodic table

Analogous to liquid drop: nucleons attracted by short range force, but do not collapse due to shorter range repulsive force

Surface of drop is well defined

Drop held together by surface tension $\propto$ Area

Binding energy $B = E_{drop} - E_{constituents}$

A drop will form if energetically favourable i.e. $E_{drop} < E_{constituents}$

Larger B means more tightly bound nucleus - more stable!

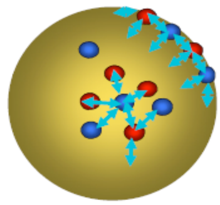
each nucleon contributes approx. same BE ⇒ $B \propto A$ (or B/A ~ const)

each nucleon only feels neighbours not all nucleons otherwise get $B \propto A(A-1)$

nuclear density is ~ constant out to surface ⇒ surface nucleons contribute less B

surface area ~ R^2 and $R \sim A^{1/3}$ therefore surface area ~ $A^{2/3}$

$A \propto$ Volume



Postulate three terms for binding energy:

volume term $\propto$ volume - nucleons are attractive & "condense" into nuclei

surface term $\propto$ area - surface nucleons less tightly bound - fewer neighbours!

coulomb term $\propto$ Z^2/radius - for uniform charge distribution within drop

What sign will each term have?

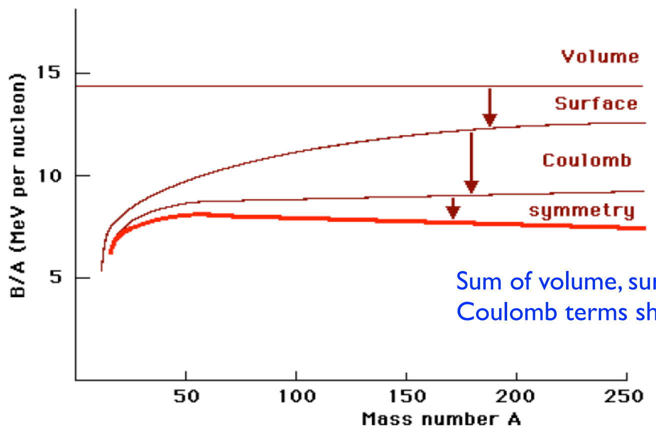
$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}}$$

Qualitatively behaviour is as expected:

- surface term: largest effect for small nuclei
- volume term is constant by construction
- Coulomb term: largest effect for high Z nuclei

a_V a_S and a_C are constants determined from the BE/A vs A curve on previous page

This formula is incomplete: predicts greatest binding energy for Z=0 (for fixed A)



Sum of volume, surface and Coulomb terms show positive slope

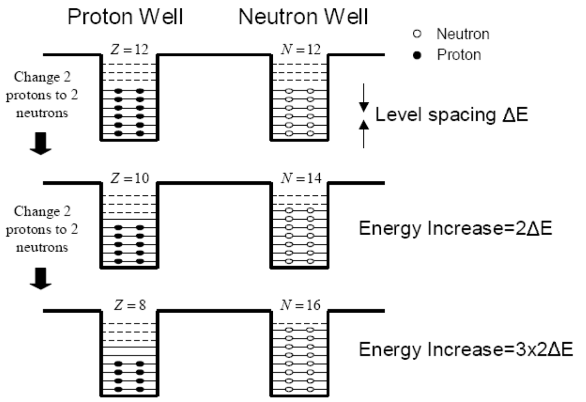
We forgot to include QM nature of nucleons symmetry!

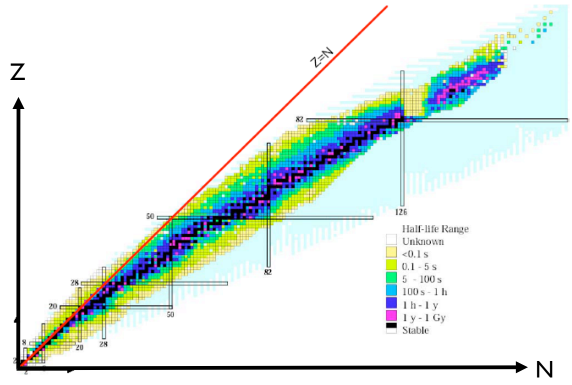
neutrons & protons are fermions ⇒ Paul Exclusion Principle

forbids identical fermions from same QM state

will influence nucleons in potential wells

ΔE is similar for neutron & proton ⇒ <u>for fixed A</u>, energy is minimised by having Z=N





Stable nuclei prefer to have Z = A/2 (i.e. Z=N)

Strongly obeyed for low Z nuclei, weakly observed for high Z

Postulate Binding Energy term ~ $-(A-2Z)^2/A$

Reduces BE when Z ≠ N quadratically

Suppressed as A increases

Experiments show that 2p or 2n are always more strongly bound than 1p+1n

Approx. 3000 have been studied - only ~240 are stable

170	stable with even N and even Z
60	stable with even N and odd Z
4	stable with odd N and odd Z

We add a pairing term the equation

- For odd A nuclei (δ=0)
  - Z even, N odd
  - Z odd, N even
- For A even
  - Z odd, N odd $(-\delta)$
  - Z even, N even $(+\delta)$

Spin effects produce the pairing term
Nucleon <u>pairs</u> with net spin = 0 are more bound
Such nucleons have very closely overlapping wave functions
They tend to be closer together - thus more bound

Pairing Term

Contribution to binding energy is

$$\delta(Z, A) = \frac{a_P}{A^{1/2}}$$

Semi Empirical Mass Formula

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

The constants are obtained by fitting data (A>20)
Light nuclei not used - additional structure due to shell closure (see later)

Given the binding energy we can calculate nuclear masses:

nuclear mass

$$M_N(A,Z) = Zm_p + (A-Z)m_n - B/c^2$$

m_p= proton mass
m_n= neutron mass

Nuclear masses are difficult to measure (need to remove all electrons!)
Much easier to compare to measurements of atomic masses
Take into account mass of electrons:

atomic mass

$$M(A,Z) \simeq Zm(^1H) + (A-Z)m_n - B/c^2$$

$m(^1H)$ =hydrogen atom mass

This is an approximation: neglecting binding energy of electrons!

Approximation is small: m_e=0.5 MeV/c^2 and electron binding energies ~ keV
Nuclear masses are ~ GeV/c^2

$a_V A$	a_V=15.56 MeV
$-a_S A^{2/3}$	a_S=17.23 MeV
$-a_C \dfrac{Z^2}{A^{1/3}}$	a_C=0.697 MeV
$-a_A \dfrac{(A-2Z)^2}{A}$	a_A=23.285 MeV
$+\delta(Z,A)$	$\delta = \begin{cases} -12.0/A^{1/2} & \text{(oo nuclei)} \\ 0 & \text{(eo/oe nuclei)} \\ +12.0/A^{1/2} & \text{(ee nuclei)} \end{cases}$

oo = odd-odd nuclei = N is odd and Z is odd
ee = even-even nuclei = N is even and Z is even

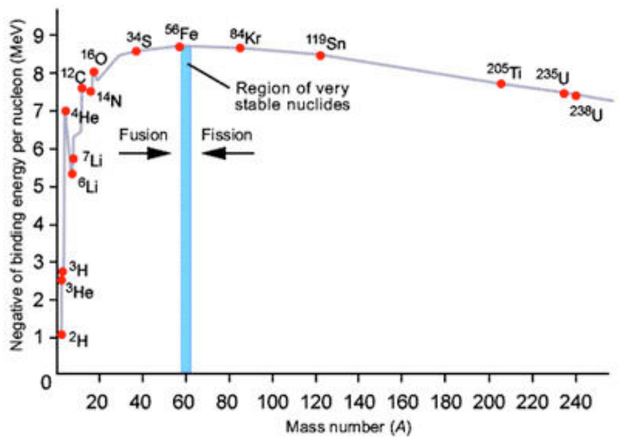
note A dependence of δ pairing term is sometimes written differently

sometimes 12.0 $A^{-1/2}$ sometimes 34.0 $A^{-3/4}$

depends on how fit to data is performed...

use values given on problem / exam sheets

no need to memorise them!



SEMF provides good description of data
works for med - high mass nuclei
Binding energy peaks for A~60
on RHS nuclei are fissile
on LHS nuclei like to undergo fusion

Slow tail to rhs is due to long range of Coulomb repulsion - protons in large A nuclei feel Coulomb replusion from all other p^+ but nuclear attraction from few neighbours

Success of formula hints suggests assumptions were meaningful

In particular that there is a short range attraction

If attractive force were long range then there would be a term prop. to A(A-1)

nuclear strong force saturates i.e. limited to short range

- The Semi Empirical Mass Formula only qualitative treatment of spin
- Assumption of spherical nucleus implies zero nuclear electric quadrupole moment
- Drop model implies rotational/vibrational states - no predictions here
- Is useful for examination of β decay
- and nuclear fission

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

Consider M for fixed A:

$$M(Z,A) \simeq Zm(^1H) + Nm_n - \left(a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z) \right)$$

quadratic in Z ⇒ parabola with min Z

Can solve for Z_{min}

Lowest mass = largest B = most stable nucleus

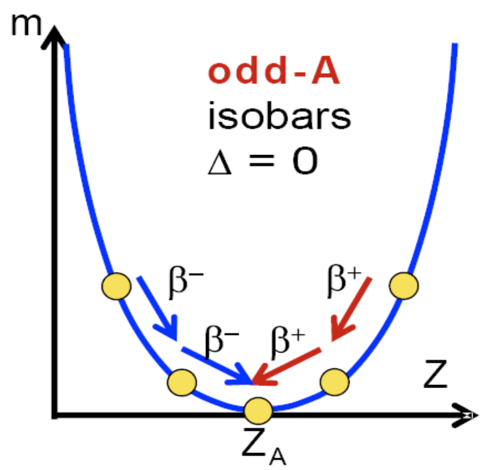
Odd A:

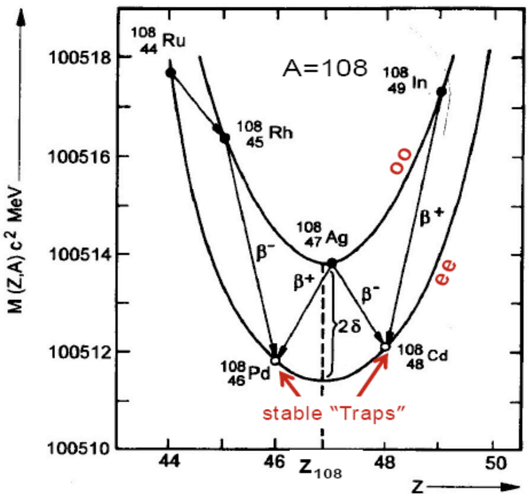
one parabola with a minimum

Z_{min} can be reached via β-decay (A is constant)

For Z > Z_{min} proton → neutron

For Z < Z_{min} neutron → proton





Even A:

Two parabolas arise from δ term separated by 2δ

$\delta = \pm 12.0/A^{1/2}$ (even: oo / ee nuclei)

β transitions switch between alternate parabolas

Two stable isobars may exist

Java applet for SEMF calculation of stability

http://www.physics.sjsu.edu/tomley/Semf.html