Nuclear Physics and Astrophysics

PHY-302

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Lecture 2 - Introduction

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Notation

Nuclides

A Nuclide is a particular nucleus and is designated by the following notation:

$${}^{A}_{Z}C_{N}$$

Z = Atomic Number (no. of Protons)
A = Atomic Mass Number (no. of Nucleons)
A = Z+N (Nucleons = Protons + Neutrons)
N = Number of Neutrons (Sometimes Omitted)

$${}^{14}_{6}C_{8}$$

14 nucleons
6 protons
8 neutrons

Nuclides with identical **Z** but different **N** are called **ISOTOPES**.
Nuclides with identical **A** are known as **ISOBARS**.
Nuclides with identical **N** are known as **ISOTONES**.
Long-lived (meta-stable) excited states of nuclei are known as **ISOMERIC**.

There are far too many nuclei to cover in such a course we will only cover a few with informative general properties

Units

In physics - use SI units:

distance:	metre
time:	second
mass:	kilogram
energy:	joule

For everyday objects and situations this works well

Handling atomic nuclei is not an everyday occurrance!
SI units can be used in nuclear physics...
...but they are cumbersome

e.g. proton mass = 1.67×10^{-27} Kg

Use a new system of units specifically for this area of physics
We are free to choose any system of units **provided we are consistent**
Never mix units!!!

Units

Distance – the fermi (fm)
1 Fermi = 10^{-15} m = 1 fm
Typical Nuclear sizes range from 1 fm to 7 fm for the largest nuclei

Time – the second (s)
Our familiar unit of time measurement
Range of nuclear timescales varies enormously:
lifetimes ~10^{-12} s (1 picosecond) up to millions of years (~10^{13} s)

Energy – the electron volt (eV)
The energy required to accelerate 1 electron through a 1V potential
1 eV = 1.602×10^{-19} J   (conversion rate is electron charge in Coulombs)
Typical nuclear energies are in MeV range (10^6)
Typical rest energies are much larger ~ GeV (10^9)

10^{-18}	atto-
10^{-15}	femto-
10^{-12}	pico-
10^{-9}	nano-
10^{-6}	micro-
10^{-3}	milli-
10^{0}	none
10^{3}	Kilo-
10^{6}	Mega-
10^{9}	Giga-
10^{12}	Tera-
10^{15}	Peta-
10^{18}	Exa-

Units

<u>Mass</u> – the atomic mass unit (u) or MeV/c^2
Defined so one **<u>atom</u>** of ^{12}C = 12 u
Since $E=mc^2$ we can switch between mass & energy as we please

One mole of ^{12}C has N_A atoms = 6.022×10^{23} atoms

0.012 Kg = N_A x 12 u → 1 u = 0.012/(N_A x 12)	= 1.66×10^{-27} Kg
Using $E=mc^2$ then, energy equivalent	= $1.66 \times 10^{-27} \times (2.99 \times 10^8)^2$
	= 1.48×10^{-10} J
Convert joules to eV: divide by electron charge	= 931.502 MeV

Then 1 u = 931.502 MeV/c^2

So, mass can be expressed as u, or in MeV/c^2

<u>You should never have to multiply any numerical result by 2.99×10^8 m/s</u>
If you do this, you are probably making an unnecessary step, or a mistake!!!

In Krane appendix C a full table of <u>atomic</u> masses is given.

Quantum Mechanics

As with all phenomena at small distances it is expected that Quantum Mechanics (QM) will prove an essential tool to help us understand and interpret nuclear process

It is assumed that your have some basic knowledge of QM (1st Year Courses)
Detailed solutions of Schrödinger Equation beyond this course
(see QMA next semester)

New topics will be covered qualitatively in the lectures.

Nucleons in the nucleus are in motion with kinetic energies of order 10 MeV comparing this with the nucleon rest energy of ~ 1 GeV so it is possible to use non-relativistic QM → Schrödinger Equation can apply in certain cases:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(r) + V(r)\cdot\Psi(r) = E\cdot\Psi(r)$$

Nuclear Physics is in general a TOUGH MANY BODY PROBLEM.
Will learn how to apply QM to understand models of Nuclear Physics

Quantum Mechanics

Quantum Mechanical Calculations will be applied to:

- α decay
- β decay*
- Shell model calculations
- Pauli Exclusion Principle*
- Quantum Statistics*
- Angular Momentum calculations*
- Decay Rate calculations*

Introductory material on QM for nuclear physics in chapter 2 of Krane.
Read this chapter to get an overview
We will not be concerned with mathematical solutions to Schrödinger eqn.

*These topics will require an understanding of QM beyond 1st Year
The techniques used for these will be covered in QMA course next semester
Not needed directly for this course!

Properties

Nuclear Properties

The list of instructions required to characterise all the interactions of a 50 nucleon nucleus would be of order $\mathbf{10^{64}}$! – We do not have the time !
For now we consider some of the more basic properties

The Nuclear Radius

Like the radius of an atom, the radius of a nucleus is not precisely defined
size of the nucleus depends on what is used to probe it. If one fires
electrons fired at the nucleus one determines the nuclear charge distribution
α particles measure the electromagnetic and strong interaction: distribution of nuclear matter

Building on the work of Rutherford who set a limit on the Nuclear radius
The original Nobel Prize winning work was done by Hofstadter

Nobel prize 1961

Rutherford Scattering

Rutherfords' famous scattering experiment founded nuclear physics

Scatter energetic α particles off gold foil

Measure angular deflection of α particles

At that time (1906) JJ Thompsons' model of atom was solid ball of electrons & protons

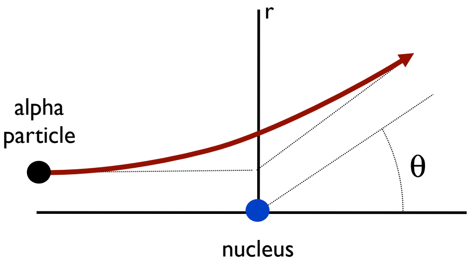
Deflections should be due to multiple interactions - many random collisions

Rutherford noticed that some collisions lead to very large deflections - rare!

Incompatible with the multiple scattering ⇒ single hard scatter

Rutherford proposed model of dense atomic nucleus and derived scattering formula

Found experiment described his model expectation



Think of this as reaction rate
as function of deflection angle.
Will define this in lecture 6

$$\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{16\pi\epsilon_0 T}\right) \frac{1}{\sin^4(\theta/2)}$$

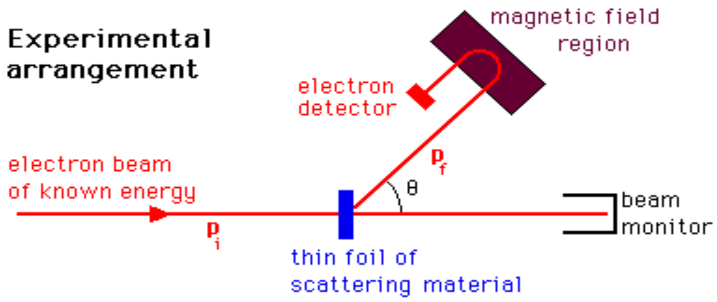
See Krane 400-401 for experimental evidence of
Rutherford Scattering

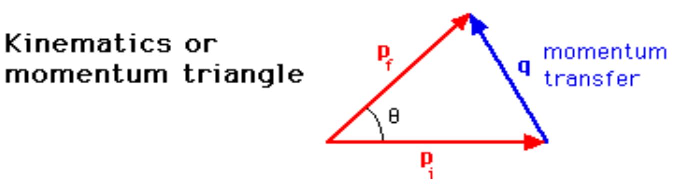
Hofstadter Experiment

Rutherford was lucky classical solution = quantum solution

But, looking inside nucleus, need probe wavelength smaller than nuclear radius
i.e. Quantum mechanics

distribution of electrons scattered from nucleus determines charge density



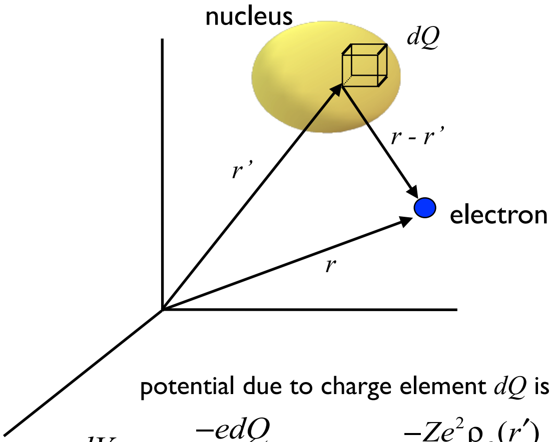


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} |F(q)|^2$$

<u>Mott Scattering</u>
Rutherford scattering
formula for point-like
electron - electron scattering

<u>Form Factor</u>
Contains all info on
charge density of
nucleus

Nuclear Radius From Hofstadter Experiment



initial electron wave function is $\psi_i \simeq e^{ik_i \cdot r}$

for particle of momentum $p_i = \hbar k_i$

Transition amplitude from initial state to final state ($k_i \to k_f$) is:

$$F(k_i, k_f) = \int \psi_f^* \cdot V(r) \cdot \psi_i \cdot dv$$

$$F(q) = \int e^{iq \cdot r} \cdot V(r) \cdot dv \qquad \text{where } q = k_i - k_f$$

the integration $V(r)$ depends on the nuclear charge density

potential due to charge element dQ is dV

$$dV = \frac{-edQ}{4\pi\varepsilon_0 |r - r'|} \qquad = \frac{-Ze^2 \rho_e(r')}{4\pi\varepsilon_0 |r - r'|} dv'$$

$\rho_e(r')$ = distribution of nuclear charge

dv' = volume element

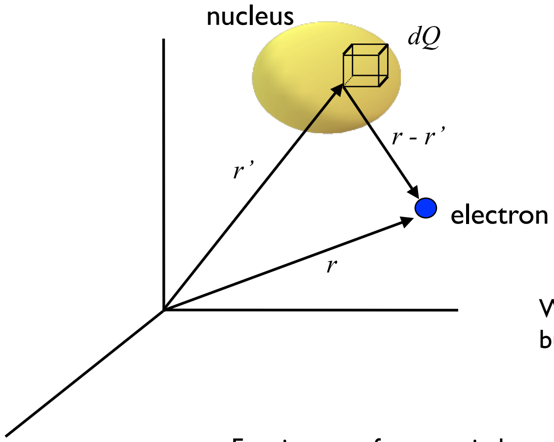
integrating over dv' gives total interaction potential

$$V(r) = \frac{-Ze^2}{4\pi\varepsilon_0} \int \frac{\rho_e(r')}{|r - r'|} dv'$$

Form factor $F(q)$ is a fourier transform of charge density

$$F(q) = \int e^{iq \cdot r'} \rho_e(r') dv'$$

therefore measure $F(q)$ and use equation to determine $\rho_e(r')$



$$F(q) = \int e^{iq \cdot r'} \rho_e(r') dv' \qquad \text{where } q = k_i - k_f$$

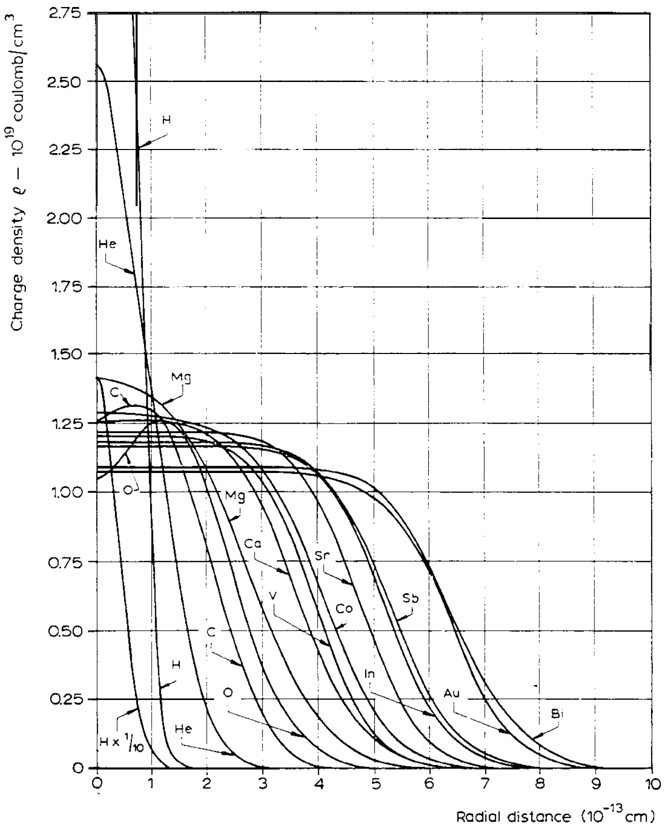
where $\rho_e(r')$ is distribution of nuclear charge

We cannot easily measure the coordinate space inside a nucleus but we can measure momentum transfer of our projectile

Fourier transforms switch one variable into another e.g. spatial co-ordinates into momentum co-ordinates

Thats what the form factor is: fourier transform of the charge distn
in terms of momentum transfer
inverse transform gives us $\rho_e(r')$ back again

http://www.nobelprize.org/nobel_prizes/physics/laureates/1961/hofstadter.html



Nuclear Charge Distribution

Results shown for several nuclei

Nucleons are not "crushed" in the centre of nucleus

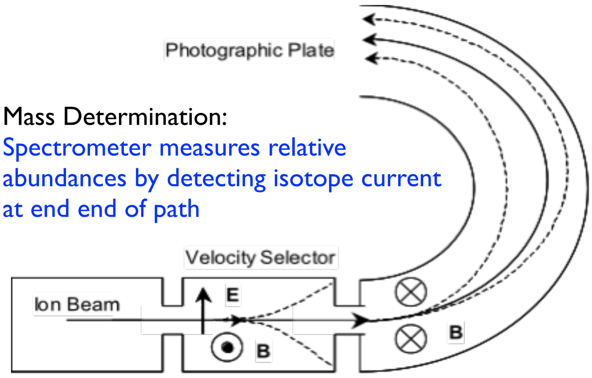
Density is approximately constant out to some surface

Hydrogen and helium behave differently to the others...

$$\frac{A}{\frac{4}{3}\pi R^3} \simeq \text{constant}$$

$$R = R_0 A^{1/3} \qquad R_0 \approx 1.2 \text{ fm}$$

Nuclear Mass Determination



Mass Determination:
Spectrometer measures relative abundances by detecting isotope current at end end of path

Mass Spectrometer:

- ionised atoms/molecules subjected to ⊥ E and B fields

  E field exerts upward force qE
  B field exerts downward force qvB

- Select E/B such that ions of particular v are selected i.e. ions undeflected when qE=qvB
- Finally uniform B field bends ions in circle with radius

  r = mv/qB

$$m = \frac{qrB^2}{E}$$

Nuclide Abundance:
Spectrometer measures relative abundances by detecting isotope current at end end of path

Isotope Separation:
Continuous running of spectrometer tuned to one mass accumulates large quantity of one isotope