NPA Homework solutions 4

26/10/09

1)

Beta$^+$ decay: $\,^{A}_{Z}X \rightarrow \,^{A}_{Z-1}Y + e^+ + \nu_e$

$$Q(\beta^+) = \left(M_N(^{A}_{Z}X) - M_N(^{A}_{Z-1}Y) - M_e\right)c^2 \quad (1)$$

M_N are <u>nuclear</u> masses

$M_e = e^-/e^+$ mass

Let M_A be atomic masses:

$$M_A(^{A}_{Z}X) = M_N(^{A}_{Z}X) + ZM_e - \sum_{i=1}^{Z} B_i^A/c^2 \quad (1)$$

B_i^A are <u>atomic electron</u> binding energies

substitute into $Q(\beta^+)$

$$Q(\beta^+) = M_A(^{A}_{Z}X)c^2 - ZM_ec^2 + \sum^{Z} B_i^A - M_A(^{A}_{Z-1}Y)c^2 + (Z-1)M_ec^2 - \sum^{Z-1} B_i^A - M_ec^2$$

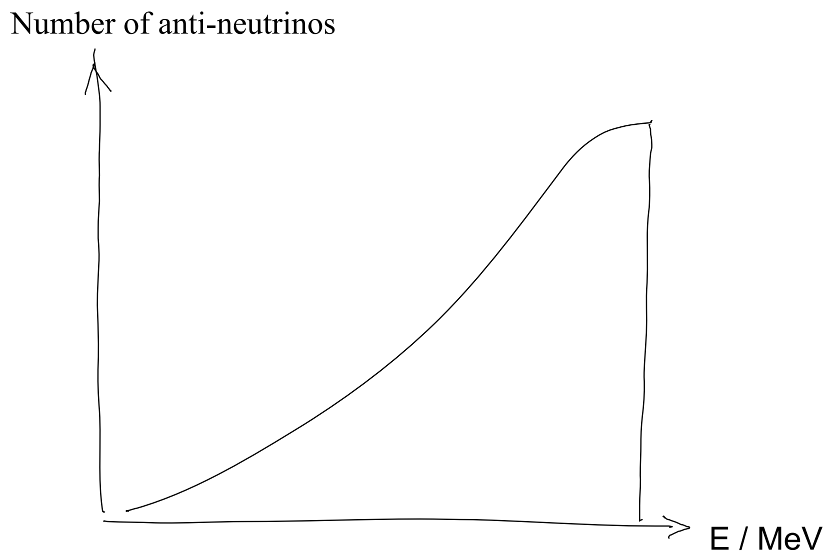
$$= M_A(^{A}_{Z}X)c^2 - M_A(^{A}_{Z-1}Y)c^2 - 2M_ec^2 \quad (1)$$

if we assume

$$\sum_{i=1}^{Z} B_i^A - \sum_{i=1}^{Z-1} B_i^A \simeq 0 \quad (1)$$

~~difference~~ Binding energy of last electron small compared to nuclear energies

2)



3)

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + \alpha$$

$Q = T_{fin} - T_{init}$ difference of initial & final kinetic energies

(1) $Q = T_\alpha - T_Y$

from energy conservation $|P_\alpha| = |P_Y|$

ie. $M_\alpha V_\alpha = M_Y V_Y$ $T_Y = \frac{1}{2} M V_Y^2$

$$\Rightarrow T_Y = \frac{M_\alpha^2 V_\alpha^2}{2M_Y} = \frac{M_\alpha}{M_Y} T_\alpha$$

$$T_\alpha = Q - \frac{M_\alpha T_\alpha}{M_Y} \quad \text{so} \quad T_\alpha = \frac{Q}{1 + M_\alpha / M_Y}$$ (2)

For $A \gg 4$ $M_\alpha \sim 4u$
$M_Y \sim Au$

$$T_\alpha = \frac{Q}{1 + \frac{4u}{Au}} = \frac{Q}{1 + 4/A}$$ (1)

use Taylor expansion $f(x) = f(x=a) + (x-a) f'(x=a) + \cdots$

$$T_\alpha = Q\left(1 - \frac{4}{A} + \frac{1}{2}\left(\frac{4}{A}\right)^2 + \cdots\right)$$

$$T_\alpha = Q\left(1 - \frac{4}{A}\right) \quad \text{for } A \gg 4$$ (2)

4)

Asymmetry term accounts for fact that nuclei prefer to have N = Z for low A. At larger A stable nuclei have a neutron excess.

Proton & neutron are distinguishable particles and fill different energy wells. Total energy minimised when N = Z for fixed A (2)

Consequence of Pauli Exclusion Principle

Pairing term accounts for observation that majority of stable nuclei are even-even and only handful are odd-odd. Pairing term is $\delta \sim +ve$ for ee nuclei, $-ve$ for eo nuclei and 0 otherwise (2)

The asymmetry term has A, Z dependance of

$$-\frac{(A-2Z)^2}{A}$$

which is zero for $Z = N$ (i.e. when $A = 2Z$).
The total binding energy is strongly decreased (quadratically) when Z & N are very different. But, this is suppressed for increasingly larger A. (1)

5) Hydrogen is the most abundant element in the universe as it is the simplest element with a nucleus of just 1 proton. The most common isotope of oxygen has 8 protons and 8 neutrons. This means that ^{16}O is **doubly magic**, i.e. both N and Z are magic numbers making this nucleus particularly stable. In other words neutrons and protons have filled complete nuclear shells. Given that plentiful supply of H and the stability of ^{16}O indicates that H_2O is an abundant substance.