NPA Homework solutions 2

16/10/10

1) $^{235}U \rightarrow {}^{231}Th + {}^{4}He$ [1]

Q = final kinetic energy – initial kinetic energy

Assuming the U and Th nuclei are at rest then the reaction Q is equal to the kinetic energy of the alpha particle = 4.678 MeV [1]

$M(^{235}U) = Q + M(^{231}Th) + M(^{4}He)$
$= 4.678 + (231.036299 + 4.002603) \times 931.502$
$= 4.678 + 218939.2 = 218943.9\ MeV/c^2$
or equivalently 235.043924 u [2]

The binding energy is given by
$B = Zm_p + Nm_n - M(^{235}U)$
$= 92 \times 1.00727647 + 143 \times 1.00866501 - 235.043924$
$= 1.8646$ u
$= 1736.9$ MeV [2]

2)

a) alpha decay conserves
- charge
- energy-momentum
- atomic nr
- atomic mass nr

b) Beta decay conserves
- charge
- energy-momentum
- atomic mass nr

c) Gamma decay conserves
- charge
- energy-momentum
- atomic nr
- atomic mass nr

3)

For ^{14}N $\quad r = r_0 A^{1/3} \quad r_0 \sim 1.2$ fm $\quad A \sim 14$

$r = 2.9$ fm

Geometric cross section of $^{14}N = \pi r^2 = 26.4$ fm^2

The interaction cross section is 0.17 fm^2

This large difference reflects the fact that the interaction cross section quantifies the dynamics & properties of the nuclear forces ie is <u>not</u> related to the geometric size of the nuclei

3)

Activity of the sample = measured activity – background

$126/3600 - 0.01 = 0.025$ Bq [1]

^{14}C has $\lambda = 1.2092 \times 10^{-4}\ y^{-1}$ thus the mean lifetime $m = 1/\lambda = 8269.9$ yr

Half-life = $\ln 2 / \lambda$

$= 5732.2$ yr

4g ^{12}C contains $4\ N_A/12$ atoms = 2×10^{23} atoms

At time of death (t=0) activity was due to one ^{14}C in every 10^{12} of ^{12}C nuclei

Thus nr ^{14}C nuclei = $10^{-12} \times 2\times10^{23} = 2\times10^{11}$ atoms of ^{14}C

Thus $N_0 = 2\times10^{11}$ and $N(t) = N_0 e^{-\lambda t}$

Activity $A(t=0) = -dN/dt = \lambda N_0 = 2\times10^{11} \times 1.2092\times10^{-4}\ yr^{-1}$

$= 24\times10^{6}\ yr^{-1} / 31.5\times10^{6}$ s/yr

$= 0.76$ Bq [2]

$A(0)/A(T) = e^{\lambda t} \rightarrow \ln(0.76) - \ln(0.025) = \lambda T$

Thus T the age of the artefact = 28.24×10^{3} yr [1]

The uncertainty in age σ_T arises from the number of counts measured and thus enters into the uncertainty on the measured activity, σ_A

$T = [\ln(0.76) - \ln A]/\lambda$

So, $\sigma_T^2 = (dT/dA)^2 \sigma_A^2$

$\sigma_T = \sigma_A/(A\lambda)$

error on number of counts is $\sqrt{126} = 11$

error on <u>total</u> activity = $11/3600 = 0.003\ s^{-1}$

error on background subtracted activity = $0.003\ s^{-1}$

and so $\sigma_T = 0.003 / (0.025 \times 1.2092\times10^{-4}\ yr^{-1}) = 992$ yr

The final answer for the age of the artefact is $28{,}000 \pm 1000$ yr. [2]

5)

$$^{197}_{79}Au + n \longrightarrow {}^{198}_{79}Au + \gamma$$

$$^{198}_{79}Au \xrightarrow{\lambda_1} {}^{198}_{80}Hg + e^- + \bar{\nu}_e$$

$\cancel{\frac{dN_1}{dt}}$ $\quad N_1 \quad\quad N_2$ $\quad\quad$ 1

$$\frac{dN_1}{dt} = P - \lambda_1 N_1 \qquad \lambda_1 = \frac{1}{3.89} = 0.257\ d^{-1}$$

$$P = 10^{10}\ s^{-1} = 8.64 \times 10^{14}\ d^{-1}$$

trial solution $\quad N_1 = Ae^{-\lambda_1 t} + B$

$$\frac{dN_1}{dt} = -\lambda_1 A e^{-\lambda_1 t} = P - \lambda_1 N_1$$

$$= P - \lambda_1 \left(Ae^{-\lambda_1 t} + B\right) \qquad 1$$

$$= P - \lambda_1 Ae^{-\lambda_1 t} - \lambda_1 B$$

$$\Rightarrow 0 = P - \lambda_1 B \quad \Rightarrow B = P/\lambda_1$$

$$@\ t=0\ N_1 = 0 : \quad N_1 = Ae^{-\lambda_1 t} + P/\lambda_1 \qquad 1$$

$$\therefore A = -P/\lambda_1$$

cont.) $N_1(t) = \frac{P}{\lambda_1}\left(1 - e^{-\lambda_1 t}\right)$ (1)

After 6 days $P = 10^{10}\,s^{-1} = 8.64 \times 10^{14}\,d^{-1}$

$\lambda_1 = 0.257\,d^{-1}$

$N_1 = 3.36 \times 10^{15}\left(1 - e^{-0.257 \times 6}\right)$

$= 2.64 \times 10^{15}$ atoms ^{198}Au (1)

How many ^{198}Hg after 6 days?

$dN_2 = +\lambda_1 N_1\, dt \implies \frac{dN_2}{dt} = \lambda_1 N_1$

$\implies \frac{dN_2}{dt} = P\left(1 - e^{-\lambda_1 t}\right)$

Need a trial sol^n.

The 2 principle time dependences are

i) exponential - from decay of N_1

ii) linear - from production N_1

Alternatively:
get $N_2(t)$ by integrating
$\int dN_2 = \int P(1-e^{-\lambda_1 t})\, dt$

∴ try a trial solution which is sum of exponential + linear

i.e. $N_2(t) = A e^{-\lambda_1 t} + Bt + C$

$\frac{dN_2}{dt} = -A\lambda_1 e^{-\lambda_1 t} + B$

equate coeffs $\implies A = P/\lambda_1 \quad B = P$ (1)

At $t=0$, $N_2 = 0 \implies C = -P/\lambda_1$

So $N_2(t) = \frac{P}{\lambda_1} e^{-\lambda t} + Pt - P/\lambda_1 = P\left[\frac{e^{-\lambda_1 t} - 1}{\lambda_1} + t\right]$ (1)

$N_2(t=6) = 2.54 \times 10^{15}$ atoms (2)

c) Equilibrium reached when $\frac{dN_1}{dt} = 0$

$$\lambda_1 N_1(t) = P$$

$$N_1(t) = P/\lambda = \underline{\underline{3.36 \times 10^{15}}} \text{ atoms}$$ (2)