

NPA Homework solutions 1

08/10/10

- 1) The neon isotope  $\text{Ne}_{11}$  has  $Z=10$ ,  $N=11$  and therefore  $A=21$  [2]
- 2) From the lecture notes and the result of the Hofstadter experiment the nuclear mass density is measured to be  $\sim 2 \times 10^{17} \text{ Kg m}^{-3}$ . Assuming the neutron star is a giant nucleus, i.e. has the same density, then the mass of  $1 \text{ cm}^3 = 0.01^3 \text{ m}^3 = 10^{-6} \text{ m}^3$  is simply  $2 \times 10^{17} \times 10^{-6} = 2 \times 10^{11} \text{ Kg (!)}$  [2]

No marks awarded if you needed to look up the mass/size of a neutron star since you didn't read the question properly. Answers within a factor of  $\sim 4$  are acceptable.

3)  $17.999160 \text{ u} = 17.999160 \times 931.502 \text{ MeV}/c^2$   
 $= 16766.25 \text{ MeV}/c^2$  [2]

The sum of nucleon masses =  $8 \times 938.280 + 10 \times 939.573 = 16901.97 \text{ MeV}/c^2$  [2]

The difference of  $135.72 \text{ MeV}/c^2$  is equivalent to about 15% of a neutron mass, or about 1% of the complete nuclear mass [0]

The mass is converted to binding energy to keep the nucleus together and is manifested in the field that keeps the nucleus bound (i.e. the strong nuclear force). [4]

a) Mass difference per nucleon =  $135.72 \text{ MeV}/c^2 / 18 = 7.54 \text{ MeV}/c^2$  [1]

Thus mass difference for  $^{235}\text{U}$  is  $7.54 \times 235 = 1771.9 \text{ MeV}/c^2$  [1]

No marks if units are incorrect!

b) The mass difference can be expressed as an energy by multiplying by  $c^2$   
 $1771.9 \text{ MeV}/c^2 \times c^2 = 1771.9 \text{ MeV}$  [2]

c) 10% of this energy is 177.2 MeV

1 mole has Avogadro's number of nuclei,  $N_A$

Total molar energy release =  $N_A \times 177.2 \text{ MeV} = 1.1 \times 10^{26} \text{ MeV}$  [4]

d) In Joules: =  $1.1 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{-6} \text{ J} = 1.8 \times 10^{13} \text{ J/mol}$  [2]

This compares to  $1.3 \times 10^6 \text{ J/mol}$  from ethanol, i.e.  $10^7$  difference!

4)

a) potential  $V(r)$  is the Coulomb potential

$$V(r) = \frac{z_1 z_2 e^2}{4\pi \epsilon_0 r} \quad z_1 = z_2 = 1$$

$$r = 1 \text{ fm}$$

from Krane  $\frac{e^2}{4\pi \epsilon_0} = 1.44 \text{ MeV fm}$

$$\therefore V = \underline{\underline{1.44 \text{ MeV}}} \quad (2)$$

b)  $z_1 = 79 \quad z_2 = 2 \quad r = 10 \text{ fm}$

$$\therefore V = 1.44 \times \frac{z_1 z_2}{10} = \frac{\cancel{114} \text{ MeV}}{\underline{\underline{22.8 \text{ MeV}}}} \quad (2)$$

5) Of course, there is no difference in decay rates, they are all the same! [2]

To convince yourselves you can compare the typical energy of nuclear processes ( $\sim 1\text{-}10 \text{ MeV}$ ) to :

- the typical thermal kinetic energy of gases at std temp/pressure
- as above but for, lets say, factor 100 higher pressure
- as above but for, lets say, factor 100 higher temperature
- the gravitational potential energy between two nuclei with surfaces touching, or even the gravitational potential between a nucleus at Earth's surface and the Earth itself.

All of these energies are far below the  $\sim \text{MeV}$  scale. We will see later that nuclear reactions can only occur when sufficient energy is supplied to cause a transition from from nuclear state to another, for example an excitation of a nucleon to a higher energy level (just like for atomic electrons). If insufficient energy is supplied then the transition is less likely to occur.

6) I forgot that the answer is actually given in the lecture notes! In general the time over which a radio-isotope can be used for dating is no more than about 10 half-lives since over this period the number of active nuclei will have reduced to 0.01% =  $(\frac{1}{2})^{10}$  of their original number making detection difficult unless the sample is very large. Thus the answer is about 50,000 years or any number within ~ factor 2. [2]

<sup>41</sup>Ar is unsuitable for several reasons:

a) the half life is far too short! It is around 100 mins only. [2]

b) Ar is inert (noble gas) and so is not readily taken up by biological organisms. [2]