Queen Mary
University of London

BSc EXAMINATION

PHY-210 Electric and Magnetic Fields

Time Allowed: 2 hours 15 minutes

Date: 15th May 2003
Time: 14:30

Answer THREE questions in total, including at least one from Section 1 and at least one from Section 2. All questions carry equal marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

Data	Permittivity constant	$\varepsilon_0 = 8.85 \times 10^{-12}$	F m^{-1}
	Permeability constant	$\mu_0 = 4\pi \times 10^{-7}$	H m^{-1}
	Electronic charge	$e = 1.60 \times 10^{-19}$	C
	Mass of electron	$m_e = 9.11 \times 10^{-31}$	kg

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SECTION 1

1

(a) What is the definition of the electric field at a point in space? Write down a vector equation for the electric field at a distance r from a point charge and define the meaning of all the symbols used. [6]

(b) The x-y plane contains point charges with magnitudes and positions as follows:

A charge of Q_1 lies at co-ordinates (0,L).

A charge of Q_1 lies at co-ordinates (0,-L).

A charge of $-Q_2$ lies at the origin.

A charge of $-Q_2$ lies at co-ordinates (2L,0) .

(i) Draw a diagram showing the position of all the charges and the three electric forces acting on the charge $-Q_2$ located at (2L,0) . [6]

(ii) The resultant force on $-Q_2$ located at (2L,0) is found to be zero. Show how this determines the charge Q_1 in terms of Q_2 . [8]

(c) The charge $-Q_2$ at the origin is removed and replaced by an electric dipole, in which the charges are separated by a negligible distance. The dipole lies along the positive y-axis with negative charge nearest to the origin. Show that if the dipole moment is P, the torque vector $\boldsymbol{\tau}$ acting on the dipole is given by

$$\boldsymbol{\tau} = -\frac{PQ_2}{16\pi\varepsilon_o L^2}\ \hat{\mathbf{k}} \quad . \qquad [10]$$

2 (a) Write down an equation for Gauss's law for the electric field and explain briefly in words, with the aid of a diagram, what the equation tells us about the relationship between electric field and charge. [8]

(b) A sphere of radius R_1 contains charge uniformly distributed throughout its volume with charge density (charge per unit volume) ρ. Show that, at a point whose distance from the centre is r, the magnitude of the electric field, E, is given by

$$E = \frac{\rho r}{3\varepsilon_0} \text{ for } r \le R_1 \qquad \text{and} \qquad E = \frac{\rho R_1^{\,3}}{3\varepsilon_0 r^2} \text{ for } r > R_1.$$

Sketch the variation of E with radial distance r, both inside and outside the sphere. [13]

(c) Calculate the work done in taking a positive test charge of 0.5C from the surface of the sphere at $r = R_1$ to the centre of the sphere at $r = 0$. (You may assume that the sphere contains positive charge.) [9]

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3 (a) Write down an equation which gives the relationship between electric field and electric potential. [5]

(b) The magnitude, E, of the electric field above a large flat conducting plate carrying surface charge density σ is $\sigma/K\varepsilon_0$ where K is the dielectric constant of the medium above the plate. A parallel plate capacitor has plates of area A separated by a distance d. The space between the plates is filled with a material of dielectric constant K. Show that the capacitance, C, is given by

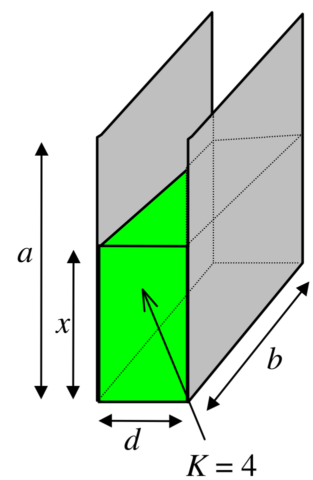
$$C = \frac{K\varepsilon_0 A}{d}.$$ [10]

(c) Hence show that the energy density, u, of the electric field between the plates is given by

$$u = \frac{1}{2} K\varepsilon_0 E^2.$$ [8]

(d) A parallel plate capacitor with rectangular plates of sides a and b and separation d is used to measure the level of a liquid whose dielectric constant is 4, as illustrated in the diagram below. Assuming that the air above the liquid has a dielectric constant of unity, show that the capacitance is given by

$$C(x) = \frac{\varepsilon_0 b}{d}(3x + a) \quad .$$



[7]

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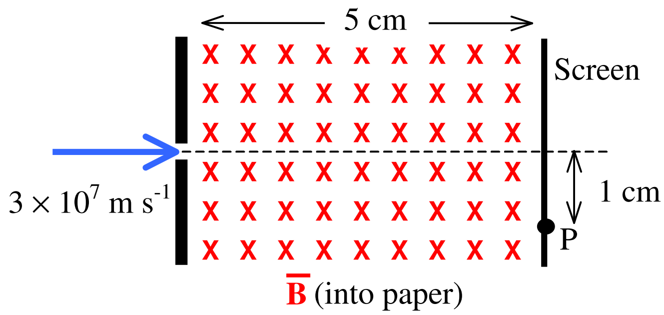
SECTION 2

4 (a) Write down the equation for the Lorentz force experienced by a particle of charge Q moving with velocity $\overline{\mathbf{v}}$ in an electric field $\overline{\mathbf{E}}$ and a magnetic field $\overline{\mathbf{B}}$. Explain why the magnetic force alone cannot change the speed of the particle. [8]

(b) Electrons are accelerated to a speed of 3×10^7 m s^{-1} before entering a region in which the magnetic field, $\overline{\mathbf{B}}$, is uniform and perpendicular to the direction of motion, as shown in the diagram opposite.



(i) Show that the path of an electron is a circular arc with radius

$$r = \frac{1.71 \times 10^{-4}}{B} \text{ m.}$$ [10]

(ii) It is required that the electron beam strike the screen at point P. Determine the necessary magnitude of the magnetic field, B.

[12]

5 (a) Write down an equation for Ampere's Law for the magnetic field and explain briefly, with the aid of a diagram, what this equation tells us about the relationship between magnetic field and current. [6]

(b) A very long straight wire carries a current I. Use Ampere's Law to show that the magnitude of the magnetic field, B, at a perpendicular distance r from the wire is given by

$$B = \frac{\mu_0 I}{2\pi r}.$$

[8]

(c) The force, **F**, on a charge Q moving with velocity **v** in a magnetic field **B** is given by **F** = Q(**v** x **B**). Use this, and the above result, to show that the magnitude, F_{12}, of the force per unit length between two very long parallel wires carrying currents I_1 and I_2 is given by

$$F_{12} = \frac{\mu_0 I_1 I_2}{2\pi a}.$$

where I_1 and I_2 are the currents in the wires and a is the distance between them. [10]

(d) Explain why the force is attractive if the currents flow in the same direction and repulsive if they flow in opposite directions. [6]

Please turn over

6 (a) Define the mutual inductance, M, between two circuits, and write down the relationship between the emf induced in one circuit and the current flowing in the other.

[6]

(b) A long solenoid of radius R_1 has n turns per unit length and carries a current I_1 . The magnetic field is uniform inside the solenoid, with a magnitude $B = \mu_0 n I_1$ and a direction parallel to the axis of the solenoid. A smaller one-turn coil of radius R_2 is placed inside the solenoid with its plane at an angle θ to the direction of the field.

(i) Prove that the mutual inductance, M, between the coil and the solenoid is given by

$$M = \mu_0 n \pi R_2^2 \sin\theta.$$

[10]

(ii) The inner coil carries a current I_2. Derive an expression for the magnitude of the torque, τ, which it experiences.

[10]

(iii) The inner coil rotates and comes to rest in a position of stable equilibrium. Sketch its final orientation with respect to the magnetic field in the solenoid.

[4]

End of Examination Paper **Dr S Thomas**