Queen Mary
University of London

BSc Examination

PH210 Electric and Magnetic Fields

Time allowed: 2 hours 15 minutes

Date: 4 May 2001
Time: 2.30 pm

Answer three questions in total, including at least one from Section 1 and at least one from Section 2.

Data:	Mass of the electron	m_e	=	9.11×10^{-31} kg
	Electronic charge	e	=	1.60×10^{-19} C

DO NOT TURN TO THE FIRST PAGE OF THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

SECTION 1

1 (a) Explain briefly the Principle of Superposition as it applies to electric forces or fields.

(b) A thin rod of length L carries positive charge with uniform charge per unit length λ. The rod lies along the x-axis with one end at the origin. Show that at a point on the x-axis a distance r from the origin (where $r > L$), the electric field, $\overline{\mathbf{E}}$, due to the rod is given by

$$\overline{\mathbf{E}} = \frac{\lambda L}{4\pi\varepsilon_0 r(r-L)}\hat{\mathbf{i}} \ .$$

What is the magnitude of $\overline{\mathbf{E}}$ for $r >> L$? Explain why this answer is of the same form as Coulomb's law for a point charge.

(c) Use the above equation and an argument based on symmetry to show that the electric field at a point *inside* the rod, a distance r from the origin (where $r > L/2$) is given by

$$\overline{\mathbf{E}} = \frac{\lambda(2r-L)}{4\pi\varepsilon_0 r(L-r)}\hat{\mathbf{i}} \ .$$

2 (a) Write down an equation for Gauss's law for the electric field and explain briefly in words, with the aid of a diagram, what the equation tells us about the relationship between electric field and charge.

(b) A sphere of radius R_1 contains charge uniformly distributed throughout its volume with charge density (charge per unit volume) ρ. Show that, at a point whose distance from the centre is r, the magnitude of the electric field, E, is given by

$$E = \frac{\rho r}{3\varepsilon_0} \text{ for } r \le R_1 \qquad \text{and} \qquad E = \frac{\rho R_1^3}{3\varepsilon_0 r^2} \text{ for } r > R_1.$$

Sketch the variation of E with radial distance r, both inside and outside the sphere.

(c) A spherical region of radius R_2 ($R_2 < R_1$), centred on the charged sphere, is removed. Write down expressions for the magnitude of the electric field at radius r in the three regions $r < R_2$, $R_2 < r < R_1$, and $r > R_1$.

3 (a) Write down an equation which gives the relationship between electric field and electric potential.

(b) The magnitude, E, of the electric field above a large flat conducting plate carrying surface charge density σ is $\sigma/K\varepsilon_0$ where K is the dielectric constant of the medium above the plate. A parallel plate capacitor has plates of area A separated by a distance d. The space between the plates is filled with a material of dielectric constant K. Show that the capacitance, C, is given by

$$C = \frac{K\varepsilon_0 A}{d}.$$

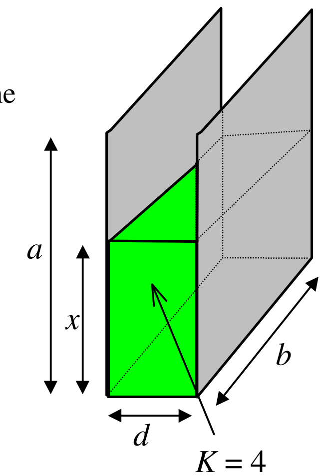
(c) Hence show that the energy density, u, of the electric field between the plates is given by

$$u = \frac{1}{2}K\varepsilon_0 E^2.$$

(d) A parallel plate capacitor with rectangular plates of sides a and b and separation d is used to measure the level of a liquid whose dielectric constant is 4, as illustrated in the diagram opposite. Assuming that the air above the liquid has a dielectric constant of unity, show that the liquid level, x, is given by

$$x = \left[\frac{4R-1}{3}\right]a$$

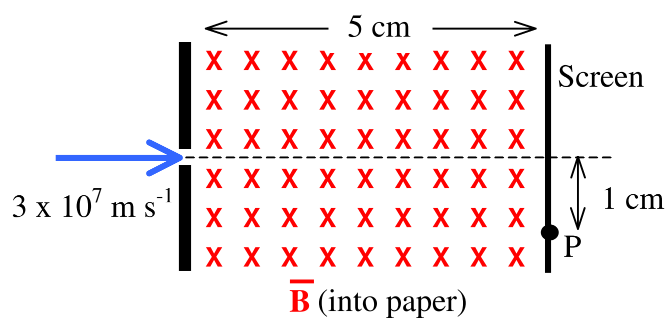
where R is the ratio of the measured capacitance to its maximum value (when $x = a$).



SECTION 2

4 (a) Write down the equation for the Lorentz force experienced by a particle of charge Q moving with velocity $\bar{\mathbf{v}}$ in an electric field $\bar{\mathbf{E}}$ and a magnetic field $\bar{\mathbf{B}}$. Explain why the magnetic force alone cannot change the speed of the particle.

(b) Electrons are accelerated to a speed of 3×10^7 m s^{-1} before entering a region in which the magnetic field, $\bar{\mathbf{B}}$, is uniform and perpendicular to the direction of motion, as shown in the diagram opposite.



(i) Show that the path of an electron is a circular arc with radius

$$r = \frac{1.71 \times 10^{-4}}{B} \quad \text{m.}$$

(ii) It is required that the electron beam strike the screen at point P. Determine the necessary magnitude of the magnetic field, B.

5 (a) Ampere's Law for the magnetic field states that $\oint \overline{\mathbf{B}} \cdot d\overline{\mathbf{L}} = \mu_0 I_{enc}$.

Explain briefly in words, with the aid of a diagram, how Ampere's law relates magnetic field and current.

(b) A very long straight solenoid has n turns of wire per unit length and carries a current I. The magnetic field is zero outside the solenoid and is uniform inside. Use Ampere's Law to show that the magnitude of the magnetic field, B, inside the solenoid is given by

$$B = \mu_0 n I.$$

(c) Maxwell's modified version of Ampere's Law states that

$$\oint \overline{\mathbf{B}} \cdot d\overline{\mathbf{L}} = \mu_0 \left[I_{enc} + \varepsilon_0 \frac{d\Phi}{dt} \right]$$ where Φ is electric flux.

Using the example of a parallel plate capacitor being charged, explain briefly and qualitatively why this modification is necessary.

6 (a) Define the mutual inductance, M, between two circuits, and write down the relationship between the emf induced in one circuit and the current flowing in the other.

(b) A long solenoid has n turns per unit length and carries a current I_1. The magnetic field inside it is uniform with magnitude $B = \mu_o n I_1$ and direction parallel to the axis of the solenoid. A smaller square coil, with a total of N turns and sides of length a, is placed inside the solenoid with its plane at an angle θ to the direction of the magnetic field.

(i) Show that the mutual inductance, M, between the coil and the solenoid is

$$M = \mu_0 n N a^2 \sin\theta.$$

(ii) The inner coil carries a current I_2. What is the magnitude of its magnetic moment? What are the two possibilities for the direction of the magnetic moment vector? Derive an expression for the magnitude of the torque, τ, which the inner coil experiences.

(iii) Calculate the work done in rotating the coil through 180^0, starting at $\theta = 90^0$. Is $\theta = 180^o$ a position of stable or unstable equilibrium? Explain your answer.

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