BSc EXAMINATION

PHY-210 Electric and Magnetic Fields

Time Allowed: 2 hours 15 minutes

Date:
Time:

Answer three questions in total, including at least one from Section 1 and at least one from Section 2. All questions carry equal marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

Data			
	Permittivity constant	$\varepsilon_0 = 8.85 \times 10^{-12}$	F m^{-1}
	Permeability constant	$\mu_0 = 4\pi \times 10^{-7}$	H m^{-1}
	Electronic charge	$e = 1.60 \times 10^{-19}$	C
	Mass of electron	$m_e = 9.11 \times 10^{-31}$	kg

DO NOT TURN TO THE FIRST PAGE OF THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

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SECTION 1

1

(a) What is the definition of the electric field at a point in space? Write down a vector equation for the electric field at a distance r from a point charge and define the meaning of all the symbols used. [5]

(b) A point charge $+Q$ lies at the coordinates (-1,0) in the x-y plane. A second point charge $-2Q$ is at coordinates (1,0). Show that the electric field **E,** at a point on the y-axis a distance d from the origin, is given by

$$\mathbf{E}_{\text{total}} = \mathbf{E}_{\text{a}} + \mathbf{E}_{\text{b}} = \frac{Q}{4\pi\varepsilon_o\left(1+d^2\right)^{3/2}}\left[3\hat{\mathbf{i}} - d\hat{\mathbf{j}}\right]$$

[14]

(c) In the system of part(b), an electric dipole, in which the charges are separated by a negligible distance, is located at coordinates (0,3). It lies along the y-axis with negative charge nearest to the origin. Show that, if the dipole moment is P, the torque vector $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = -\frac{3PQ}{4\pi\varepsilon_{\text{o}}10^{3/2}}\hat{\mathbf{k}}$$

[8]

(d) As viewed from a point above the x-y plane, in which sense (clockwise or anti-clockwise) will the electric field tend to rotate the dipole. Explain your answer.

[3]

2

(a) Explain why each of the following statements is true for a perfect conductor in electrostatic equilibrium.

(i) The electric field inside the conductor is zero. [4]

(ii) At the surface of the conductor, the electric field is everywhere perpendicular to the surface. [4]

(iii) All excess charge on the conductor is located at its surface.

[4]

(b) A very long insulating cylinder of radius R contains charge uniformly distributed inside it with charge density (charge per unit volume) ρ. Using Gauss's law show that the magnitude of the electric field E, at a perpendicular distance r from the axis of the cylinder, is given by:-

Question continues on the next page.

$$E(r) = \frac{\rho r}{2\varepsilon_o} \quad \text{for} \quad r < R \qquad \text{and} \qquad E(r) = \frac{\rho R^2}{2\varepsilon_o r} \quad \text{for} \quad r > R$$

Sketch the variation of E with r. [14]

(c) The electric potential is defined to be zero on the axis of the cylinder. Derive an expression for the potential at the surface of the cylinder (r=R).

[4]

3

(a) Write down an equation which expresses the electric field **E** as the gradient of the electric potential V.

[3]

(b) In a certain region of space, the electric potential V at a point (x, y, z) is given by

$$V(x, y, z) = (x^2z + 2y^2z - 3z^3) \text{ V}, \quad \text{where } x, y \text{ and } z \text{ are in meters.}$$

(i) Find the electric field $\mathbf{E}(x, y, z)$ as a function of position . [5]

(ii) A negative ion with charge $-2e$, where e is the electronic charge, is moved from point (2,1,2) to the origin. Find the change in the electric potential energy of the ion. Does external work need to be done to move the ion or is this work provided by the electric field? Explain your answer. [5]

(iii) An electron is released from rest at coordinates (0,0,1). Find the magnitude and direction of its initial acceleration. [5]

(iv) Show that the equipotentials in any plane parallel to the x-y plane are ellipses.

[5]

(v) A small electric dipole with equal and opposite charges of ± 2 μC and separation 1 μm is located at position (1,1,1). The dipole moment vector points in the +z direction. Find an expression for the torque acting on the dipole.

[7]

SECTION 2

4

(a) Write down the equation for the Lorentz force experienced by a particle of charge Q moving with a velocity **v** in a magnetic field **B** . [4]

(b) A particle with positive charge Q is projected with velocity **v** along the +y-axis into a region where a magnetic field **B** of strength 3T points along the –z-axis and an electric field **E** of strength 6 x10^7 V m^{-1} points along the +x-axis. The particle remains undeflected, continuing to move along the +y-axis.

(i) Draw a diagram showing the vectors **v**, **E** and **B** and the directions of the electric and magnetic forces acting on the particle. [8]

(ii) Show that the speed v of the particle is 2 x10^7 m s^{-1}. [5]

Question continues on the next page.

(c) Explain why, when the electric field is switched off, the particle describes a circular trajectory in the x-y plane. [8]

(d) The radius of the circular orbit in (c) is found to be 15mm. What is the charge to mass ratio of the particle? [5]

5

(a) Ampere's Law for magnetic fields states that $\oint \mathbf{B} \cdot d\mathbf{L} = \mu_o I$

Define each of the symbols used in this equation and explain with use of a diagram what this equation tells us about the relationship between electric current and magnetic field.

[5]

(b) A cylindrical conductor of radius 1.5mm carries a current of 20 A which is uniformly distributed over its cross sectional area. Use Ampere's Law to find the magnetic field at a perpendicular distance r from the axis of the conductor for (i) r < 1.5mm and (ii) r > 1.5mm. Sketch the variation of the magnetic field with radial distance from the axis.

[15]

(c) Maxwell's modification of Ampere's Law state that

$$\oint \mathbf{B} \cdot d\mathbf{L} = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi}{dt}$$

Using the example of a parallel plate capacitor, explain briefly why such a modification is necessary. [10]

6

(a) Define the self-inductance , L, of a circuit and write down the expression relating the induced emf to the current in the circuit. Explain how Lenz's law is used to determine the sense of the induced emf. [8]

(b) Prove that the energy stored by an inductor L carrying a current I is given by

$$U = ½\, L\, I^2 \qquad [7]$$

Where is the stored energy located? [2]

(c) A coil has a self inductance of 4 mH. An oscillating emf E is applied to the coil, where

E = 3cos (100πt) V where t is the time in seconds.

Question continues on the next page.

Derive expressions for :-

(i) the current in the coil, $i(t)$ [4]

(ii) the magnetic flux through the coil, $\Psi(t)$. [3]

(iii) the energy stored in the coil, $U(t)$. [3]

Sketch the quantities E(t), $i(t)$ and $U(t)$ over a time period of 1 cycle of the emf. [3]

End of Examination Paper **Dr S Thomas**

